Probing collision-induced electronic entanglement in ballistic conductors

M. Paulet, G. Rebora, P. Degiovanni (ENS Lyon)
M. Bienioszek, A. Feller (Université Lille)
G. Ménard, G. Fève (ENS Paris)

QUANTUMatter 2026, Barcelona

Introduction

Introduction

- Single-electron tomography through first-order electronic coherence
  - *Grenier et al., New J. Phys.* **13**, *093007 (2011)*
  - *Bisognin et al., Nat. Comm.* **10**, *3379 (2019)*
  - *Roussel et al., PRX Quantum* **2**, *020314 (2021)*

Introduction

- Single-electron tomography through first-order electronic coherence
  - *Grenier et al., New J. Phys.* **13**, *093007 (2011)*
  - *Bisognin et al., Nat. Comm.* **10**, *3379 (2019)*
  - *Roussel et al., PRX Quantum* **2**, *020314 (2021)*
- Entanglement witness transposable to second-order electronic coherence
  - *Wölk et al., Phys. Rev. A* **90**, *022315 (2014)*

Introduction

- Single-electron tomography through first-order electronic coherence
  - *Grenier et al., New J. Phys.* **13**, *093007 (2011)*
  - *Bisognin et al., Nat. Comm.* **10**, *3379 (2019)*
  - *Roussel et al., PRX Quantum* **2**, *020314 (2021)*
- Entanglement witness transposable to second-order electronic coherence
  - *Wölk et al., Phys. Rev. A* **90**, *022315 (2014)*
- Yesterday: P. Degiovanni's talk on two-electron photo-assisted tomography protocol
  - *Characterization of electronic entanglement via photo-assisted filtering*
    P. Degiovanni, Plenary session, 27/04, 10h10

Introduction

- Accessing two-electron coherence $\Longrightarrow$ probing electronic entanglement

Introduction

- Accessing two-electron coherence $\Longrightarrow$ probing electronic entanglement
- Theoretical collision model between two electronic wavepackets

Introduction

- Accessing two-electron coherence $\implies$ probing electronic entanglement
- Theoretical collision model between two electronic wavepackets
  - pure state wavefunctions: Landau wavepackets (lorentzian energy distribution)

Introduction

- Accessing two-electron coherence $\Longrightarrow$ probing electronic entanglement
- Theoretical collision model between two electronic wavepackets
  - pure state wavefunctions: Landau wavepackets (lorentzian energy distribution)
  - coherent collision: no energy leaks to the environment

Introduction

- Accessing two-electron coherence $\Longrightarrow$ probing electronic entanglement
- Theoretical collision model between two electronic wavepackets
  - pure state wavefunctions: Landau wavepackets (lorentzian energy distribution)
  - coherent collision: no energy leaks to the environment
  - high-energy electrons: no perturbation of the Fermi sea

Outline

1. Collision model and electronic coherence

2. Second-order coherence of Landau wavepackets

3. Entanglement generation and detection

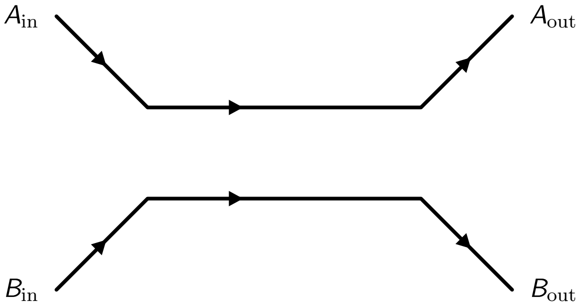
Outline

1. Collision model and electronic coherence
2. Second-order coherence of Landau wavepackets
3. Entanglement generation and detection

1.1. Interaction model for two-electron scattering

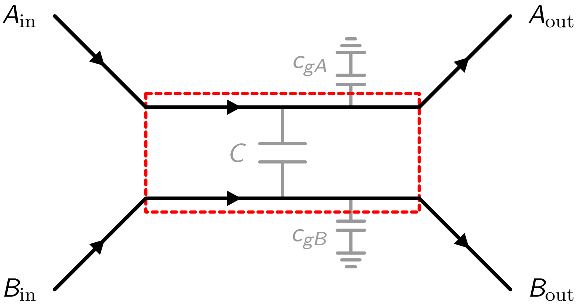
1.1. Interaction model for two-electron scattering

- Two 1D co-propagating and ballistic channels



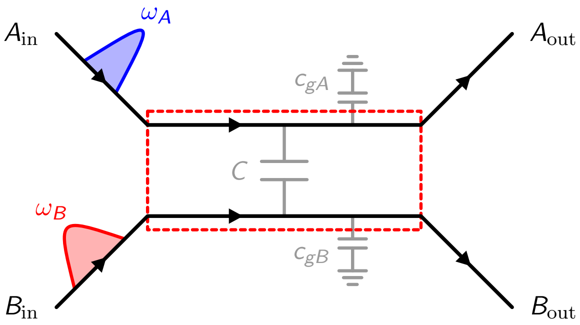
1.1. Interaction model for two-electron scattering

- Two 1D co-propagating and ballistic channels
- Coulomb interaction: unscreened capacitive coupling



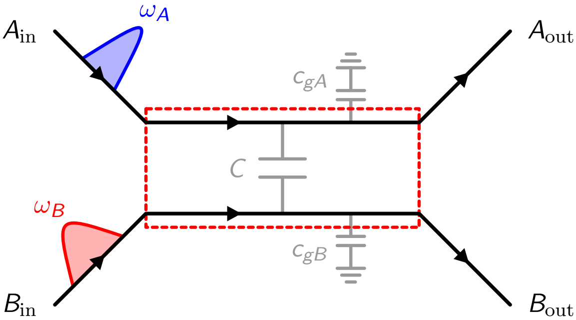
1.1. Interaction model for two-electron scattering

- Two 1D co-propagating and ballistic channels
- Coulomb interaction: unscreened capacitive coupling



1.1. Interaction model for two-electron scattering

- Two 1D co-propagating and ballistic channels
- Coulomb interaction: unscreened capacitive coupling
- Phase accumulation $\Longrightarrow$ energy transfers between wavepackets



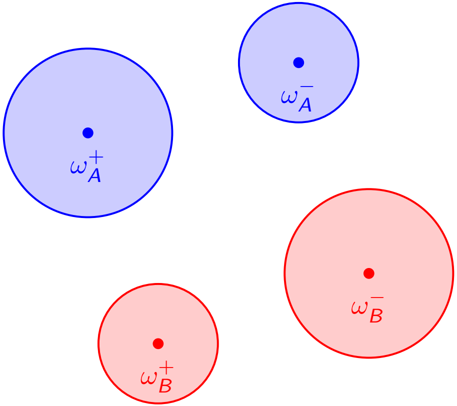
1.2. Energy-domain structure of the second-order coherence

1.2. Energy-domain structure of the second-order coherence

- $\widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^+, \omega_B^+ \,|\, \omega_A^-, \omega_B^-)$: 4D function

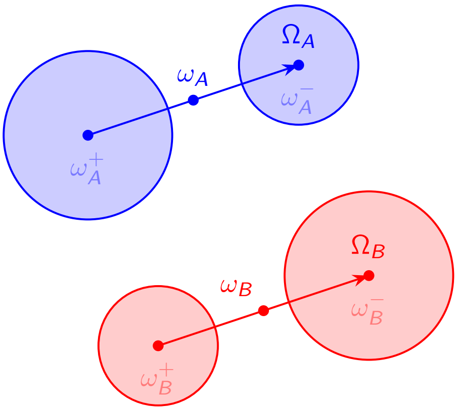
1.2. Energy-domain structure of the second-order coherence

- $\widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^+, \omega_B^+ \,|\, \omega_A^-, \omega_B^-)$: 4D function



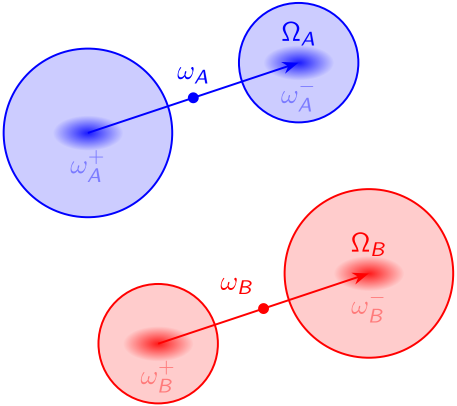
1.2. Energy-domain structure of the second-order coherence

- $\widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^+, \omega_B^+ \,|\, \omega_A^-, \omega_B^-)$: 4D function
- Classical ($\omega_{A/B}$) and quantum ($\Omega_{A/B}$) variables for each channel



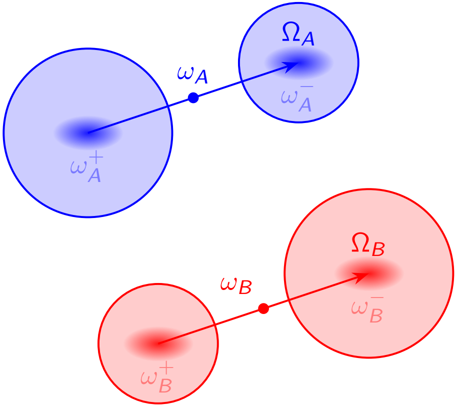
1.2. Energy-domain structure of the second-order coherence

- $\widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^+, \omega_B^+ \,|\, \omega_A^-, \omega_B^-)$: 4D function
- Classical ($\omega_{A/B}$) and quantum ($\Omega_{A/B}$) variables for each channel



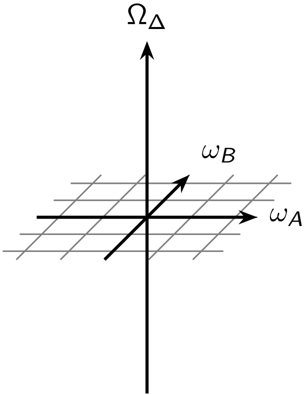
1.2. Energy-domain structure of the second-order coherence

- $\widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^+, \omega_B^+ \,|\, \omega_A^-, \omega_B^-)$: 4D function
- Classical ($\omega_{A/B}$) and quantum ($\Omega_{A/B}$) variables for each channel
- Center of mass (Ω_Σ) and relative particle (Ω_Δ) for quantum variables



1.2. Energy-domain structure of the second-order coherence

- $\widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^+, \omega_B^+ \,|\, \omega_A^-, \omega_B^-)$: 4D function
- Classical ($\omega_{A/B}$) and quantum ($\Omega_{A/B}$) variables for each channel
- Center of mass (Ω_Σ) and relative particle (Ω_Δ) for quantum variables
- Periodic source: $\Omega_\Sigma = 2\pi n f\,, n \in \mathbb{N}$
  $n = 0 \implies$ 3D-representation ($\Omega_\Sigma = 0$)



Outline

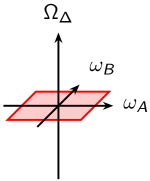
2.1. Diagonal coherence

2.1. Diagonal coherence

- Diagonal coherence $\iff$ energy probability distribution

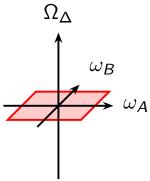
2.1. Diagonal coherence

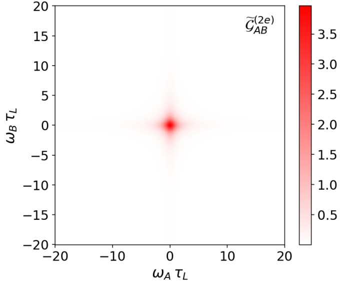
- Diagonal coherence $\Longleftrightarrow$ energy probability distribution



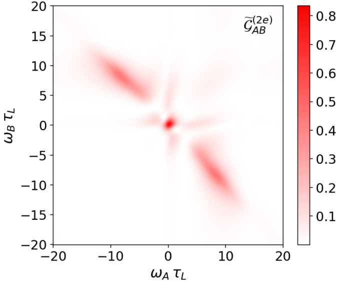
2.1. Diagonal coherence

- Diagonal coherence $\Longleftrightarrow$ energy probability distribution



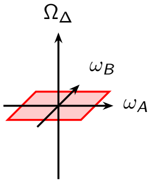


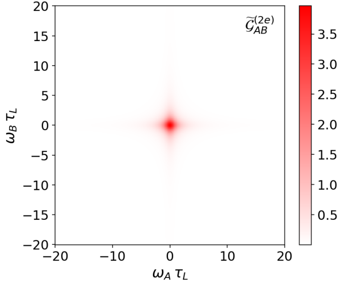
Collision $\longrightarrow$



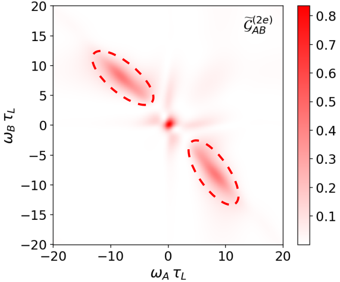
2.1. Diagonal coherence

- Diagonal coherence $\iff$ energy probability distribution
- Lateral lobes: energy transfers $A \to B$ and $B \to A$



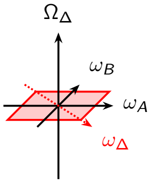


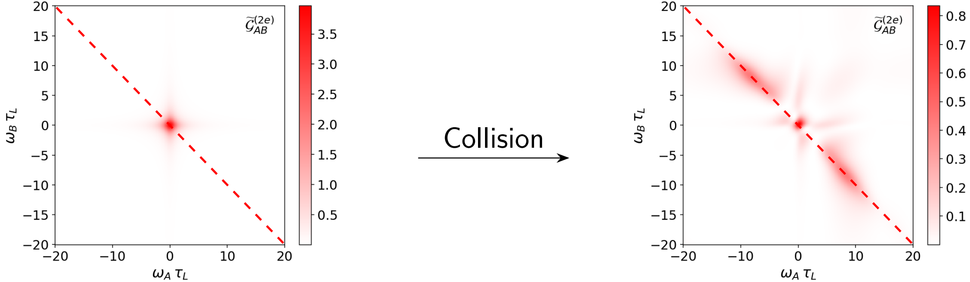
Collision



2.1. Diagonal coherence

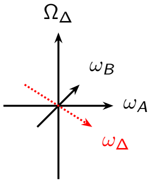
- Diagonal coherence $\iff$ energy probability distribution
- Lateral lobes: energy transfers $A \to B$ and $B \to A$

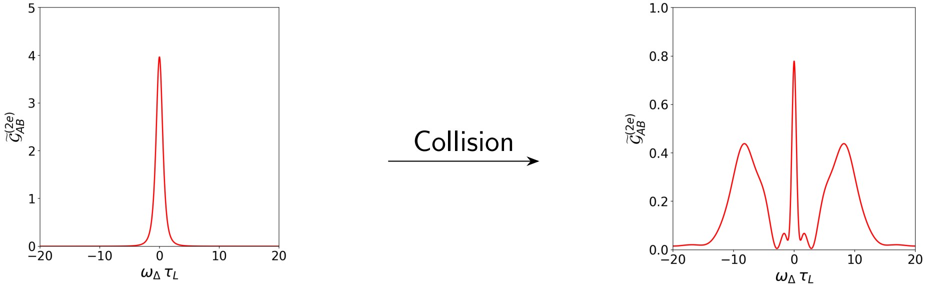




2.1. Diagonal coherence

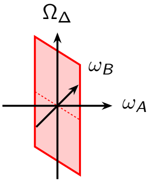
- Diagonal coherence $\iff$ energy probability distribution
- Lateral lobes: energy transfers $A \to B$ and $B \to A$



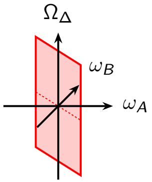


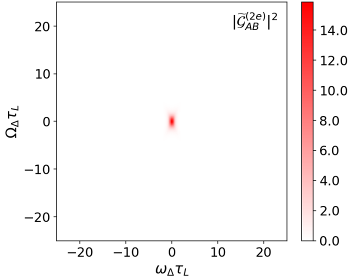
2.2. Off-diagonal coherence

2.2. Off-diagonal coherence

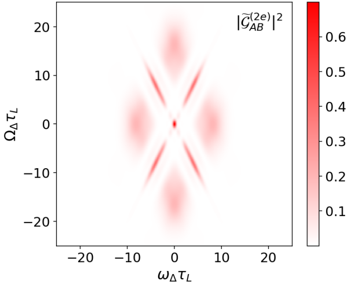


2.2. Off-diagonal coherence

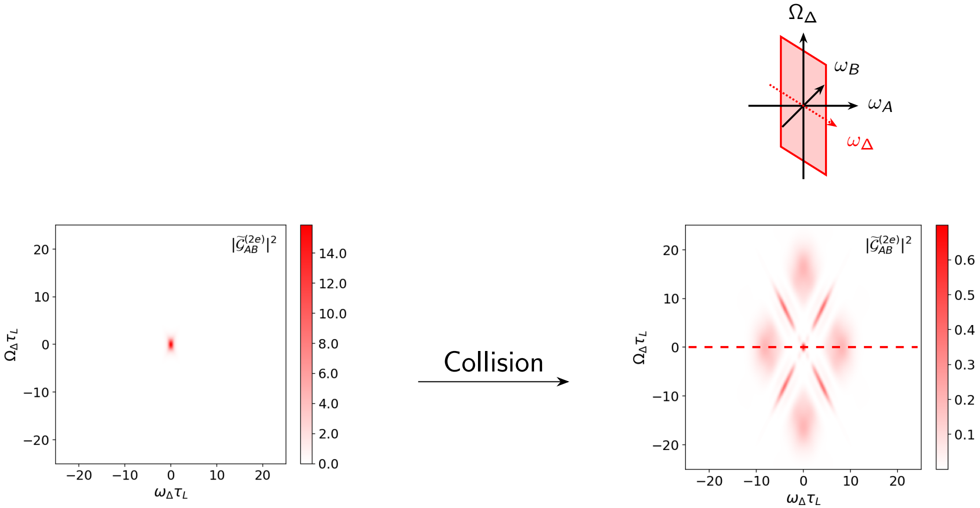




Collision →

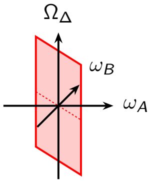


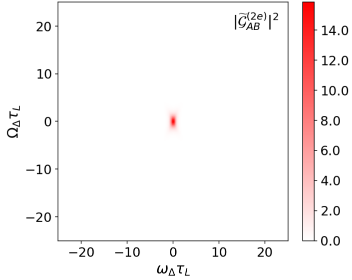
2.2. Off-diagonal coherence



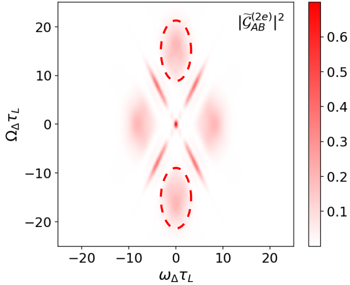
2.2. Off-diagonal coherence

- Off-diagonal lobes: interferences between lateral lobes



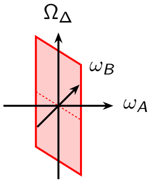


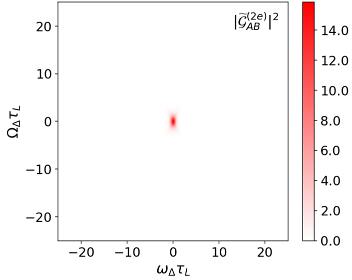
Collision



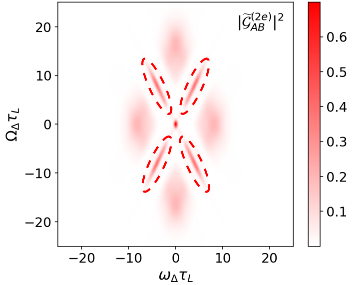
2.2. Off-diagonal coherence

- Off-diagonal lobes: interferences between lateral lobes
- Fringes: interferences central structure / lateral lobes





Collision →



Outline

3.1. Cauchy-Schwarz entanglement witness

3.1. Cauchy-Schwarz entanglement witness

$$|\widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^+,\omega_B^+\,|\,\omega_A^-,\omega_B^-)|^2 \leq \widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^+,\omega_B^-\,|\,\omega_A^+,\omega_B^-)\,\widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^-,\omega_B^+\,|\,\omega_A^-,\omega_B^+)$$

3.1. Cauchy-Schwarz entanglement witness

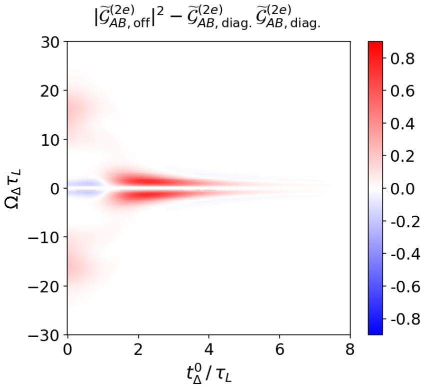
$$|\widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^+, \omega_B^+ \,|\, \omega_A^-, \omega_B^-)|^2 \leq \widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^+, \omega_B^- \,|\, \omega_A^+, \omega_B^-)\, \widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^-, \omega_B^+ \,|\, \omega_A^-, \omega_B^+)$$

- Witness $\neq$ criterion: useful only if violated

3.1. Cauchy-Schwarz entanglement witness

$$|\widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^+,\omega_B^+\,|\,\omega_A^-,\omega_B^-)|^2 \leq \widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^+,\omega_B^-\,|\,\omega_A^+,\omega_B^-)\,\widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^-,\omega_B^+\,|\,\omega_A^-,\omega_B^+)$$

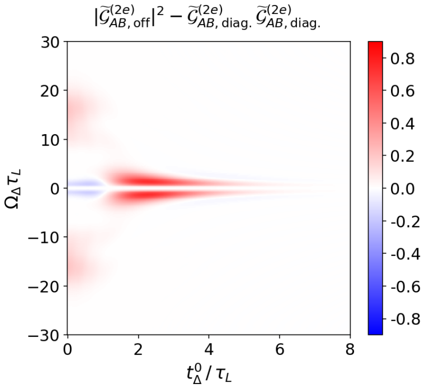
- Witness $\neq$ criterion: useful only if violated
- $t_\Delta^0 = t_A^0 - t_B^0$: offset between injection times



3.1. Cauchy-Schwarz entanglement witness

$$|\widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^+,\omega_B^+\,|\,\omega_A^-,\omega_B^-)|^2 \leq \widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^+,\omega_B^-\,|\,\omega_A^+,\omega_B^-)\,\widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^-,\omega_B^+\,|\,\omega_A^-,\omega_B^+)$$

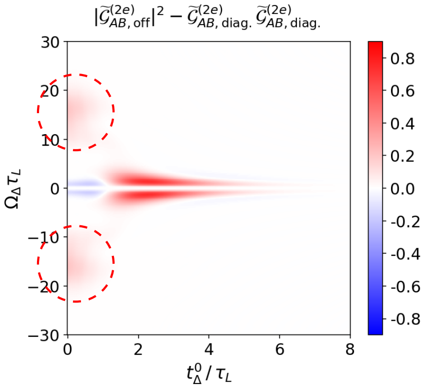
- Witness $\neq$ criterion: useful only if violated
- $t_\Delta^0 = t_A^0 - t_B^0$: offset between injection times
- Two violation regions:



3.1. Cauchy-Schwarz entanglement witness

$$|\widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^+,\omega_B^+\,|\,\omega_A^-,\omega_B^-)|^2 \leq \widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^+,\omega_B^-\,|\,\omega_A^+,\omega_B^-)\,\widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^-,\omega_B^+\,|\,\omega_A^-,\omega_B^+)$$

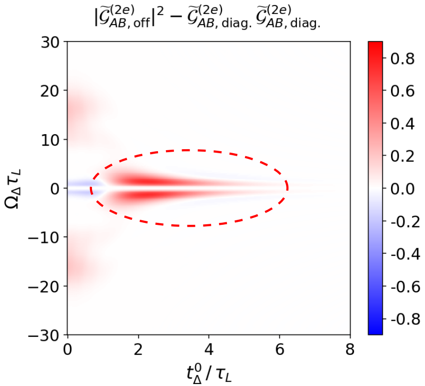
- Witness $\neq$ criterion: useful only if violated
- $t_\Delta^0 = t_A^0 - t_B^0$: offset between injection times
- Two violation regions:
  - off-diagonal clusters for $t_\Delta^0 \simeq 0$



3.1. Cauchy-Schwarz entanglement witness

$$|\widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^+,\omega_B^+\,|\,\omega_A^-,\omega_B^-)|^2 \leq \widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^+,\omega_B^-\,|\,\omega_A^+,\omega_B^-)\,\widetilde{\mathcal{G}}^{(2e)}_{AB}(\omega_A^-,\omega_B^+\,|\,\omega_A^-,\omega_B^+)$$

- Witness $\neq$ criterion: useful only if violated
- $t_\Delta^0 = t_A^0 - t_B^0$: offset between injection times
- Two violation regions:
  - off-diagonal clusters for $t_\Delta^0 \simeq 0$
  - nearly-diagonal clusters for $t_\Delta^0 \in [\tau_L, 6\tau_L]$



3.2. Time-frequency representation

3.2. Time-frequency representation

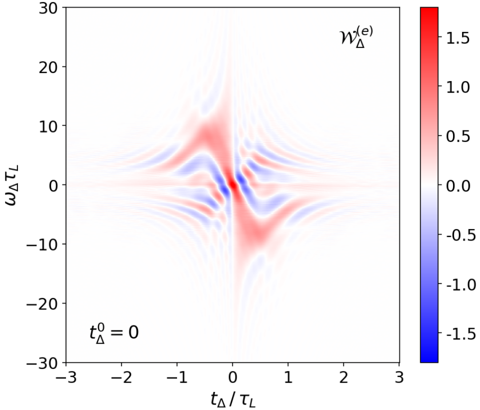
- Wigner function for the relative particle

$$\mathcal{W}_{\Delta}^{(e)}(t_\Delta \,|\, \omega_\Delta) = \int_{\mathbb{R}} dt_\Sigma \, \mathcal{W}_{AB}^{(2e)}(t_\Sigma, t_\Delta \,|\, \omega_\Sigma, \omega_\Delta)\Big|_{\omega_\Sigma^0}$$

3.2. Time-frequency representation

- Wigner function for the relative particle

$$\mathcal{W}_{\Delta}^{(e)}(t_\Delta \,|\, \omega_\Delta) = \int_{\mathbb{R}} dt_\Sigma \, \mathcal{W}_{AB}^{(2e)}(t_\Sigma, t_\Delta \,|\, \omega_\Sigma, \omega_\Delta)\Big|_{\omega_\Sigma^0}$$

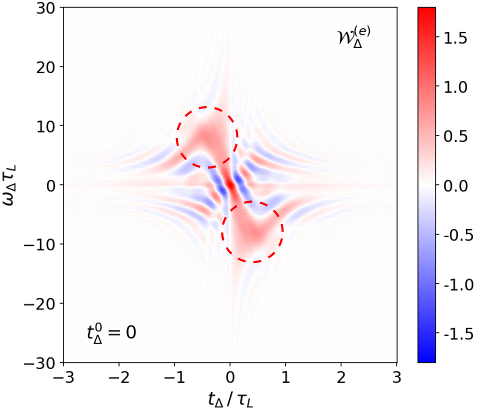


3.2. Time-frequency representation

- Wigner function for the relative particle

$$\mathcal{W}_{\Delta}^{(e)}(t_\Delta \,|\, \omega_\Delta) = \int_{\mathbb{R}} dt_\Sigma \, \mathcal{W}_{AB}^{(2e)}(t_\Sigma, t_\Delta \,|\, \omega_\Sigma, \omega_\Delta)\Big|_{\omega_\Sigma^0}$$

- Positive lobes: time of relative particle selects energy transfers



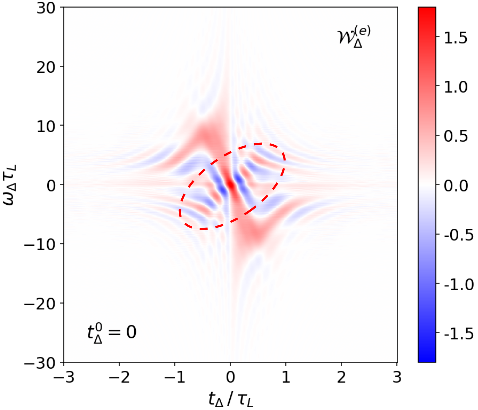
3.2. Time-frequency representation

- Wigner function for the relative particle

$$\mathcal{W}_{\Delta}^{(e)}(t_\Delta \,|\, \omega_\Delta) = \int_{\mathbb{R}} dt_\Sigma \, \mathcal{W}_{AB}^{(2e)}(t_\Sigma, t_\Delta \,|\, \omega_\Sigma, \omega_\Delta)\Big|_{\omega_\Sigma^0}$$

- Positive lobes: time of relative particle selects energy transfers

- Interference fringes: coherent superposition of exchange processes



3.2. Time-frequency representation

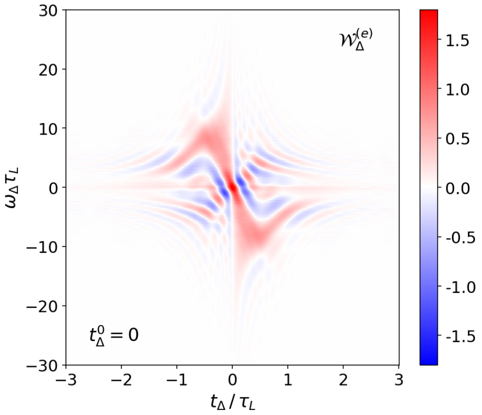
- Wigner function for the relative particle

$$\mathcal{W}_{\Delta}^{(e)}(t_\Delta \,|\, \omega_\Delta) = \int_{\mathbb{R}} dt_\Sigma \, \mathcal{W}_{AB}^{(2e)}(t_\Sigma, t_\Delta \,|\, \omega_\Sigma, \omega_\Delta)\Big|_{\omega_\Sigma^0}$$

- Positive lobes: time of relative particle selects energy transfers

- Interference fringes: coherent superposition of exchange processes

- Negativities $\Longleftrightarrow$ quantum effects



Conclusion

Conclusion

- Very simple model, but already a lot of interesting physics

Conclusion

- Very simple model, but already a lot of interesting physics
- Cauchy-Schwarz witness $\implies$ detection of collision-induced entanglement

Conclusion

- Very simple model, but already a lot of interesting physics
- Cauchy-Schwarz witness $\implies$ detection of collision-induced entanglement
- Methods and formalism can be generalized to more realistic models

Conclusion

- Very simple model, but already a lot of interesting physics
- Cauchy-Schwarz witness $\implies$ detection of collision-induced entanglement
- Methods and formalism can be generalized to more realistic models
    - decoherence, exchanges with the environment, low-energy wavepackets

Conclusion

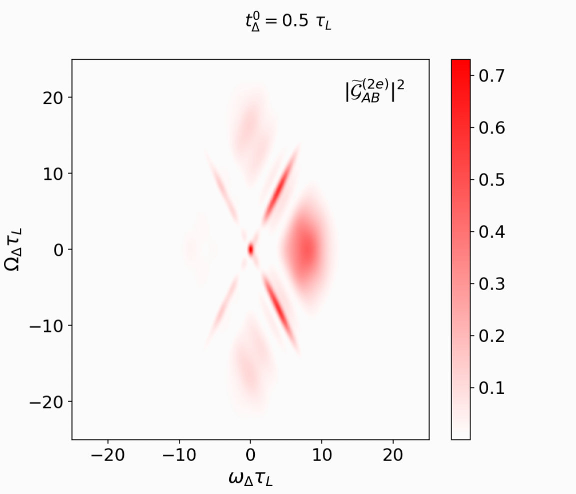
- Very simple model, but already a lot of interesting physics
- Cauchy-Schwarz witness $\implies$ detection of collision-induced entanglement
- Methods and formalism can be generalized to more realistic models
  - decoherence, exchanges with the environment, low-energy wavepackets
- How can one quantify the amount of created entanglement?

Conclusion

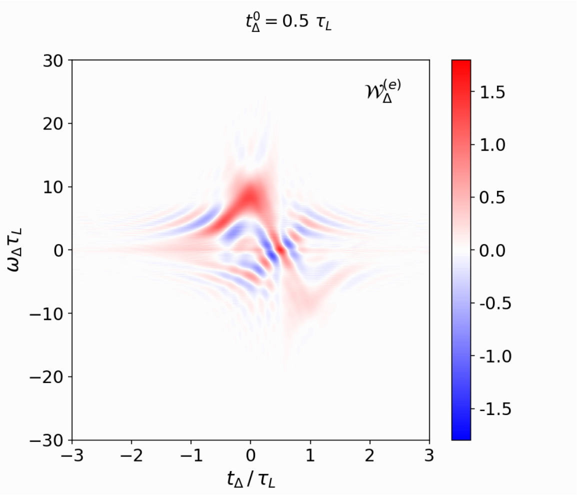
- Very simple model, but already a lot of interesting physics
- Cauchy-Schwarz witness $\Longrightarrow$ detection of collision-induced entanglement
- Methods and formalism can be generalized to more realistic models
  - decoherence, exchanges with the environment, low-energy wavepackets
- How can one quantify the amount of created entanglement?
- Only one aspect of flying qubits dynamics: part of a more general approach!
  - *From fermionic entanglement to electronic flying qubits*
    A. Feller, Workshop 2, 28/04, 15h45

Thank you for your attention!

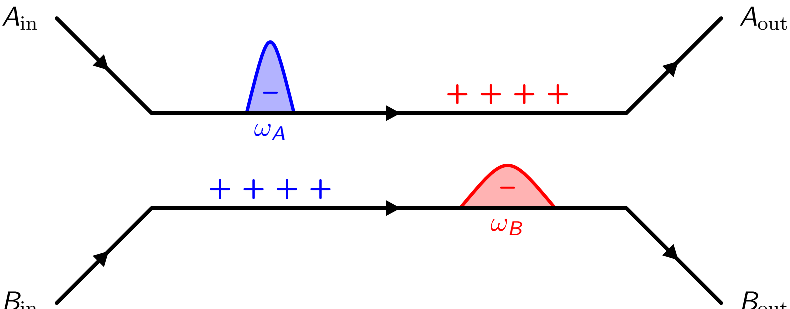
Appendix: $\widetilde{\mathcal{G}}_{AB}^{(2e)}$ function evolution with injection-time offset



Appendix: $\mathcal{W}_\Delta^{(e)}$ function evolution with injection-time offset

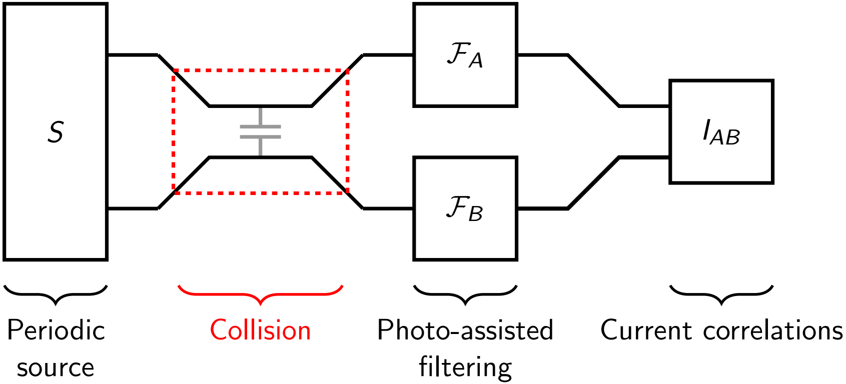


Appendix: energy transfers evolution with injection-time offset



$$t_\Delta^0 = t_A^0 - t_B^0 > 0$$

Appendix: principle of the two-electron tomography protocol



Appendix: expected experimental signal