Two-electron photo-assisted tomography protocol

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EQUBITFLY General Assembly Meeting, Lyon

AI-free presentation :)

Introduction

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- Reconstruct single-particle states *via* first-order coherence $\Longrightarrow$ $\mathcal{G}^{(e)}$ tomography
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  - *Paulet et al., arXiv:2610.12286 [cond-mat.mes-hall]*
- $\mathcal{G}^{(2e)}$ tomography to probe entanglement $\rightarrow$ see P. Degiovanni's talk!
  - *Wölk et al., Phys. Rev. A* **90**, *022315 (2014)*

Outline

1. Second-order electronic coherence
1.1 Definition and representations
1.2 Three-dimensional energy space

2. Photo-assisted two-electron tomography
2.1 From the source to the experimental signal
2.2 From the experimental signal to the coherence

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- $\widetilde{\mathcal{G}}^{(2e)}$ provides information on the nature of the excitations (their content in energy)

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$$\mathcal{W}_{AB}^{(2e)}\left(t_A, \omega_A \,|\, t_B, \omega_B\right) = \int_{\mathbb{R}^2} \frac{d\Omega_A \, d\Omega_B}{\left(2\pi v_F\right)^2} \widetilde{\mathcal{G}}^{(2e)}\left(\omega_A, \Omega_A \,|\, \omega_B, \Omega_B\right) e^{-i(\Omega_A t_A + \Omega_B t_B)}$$

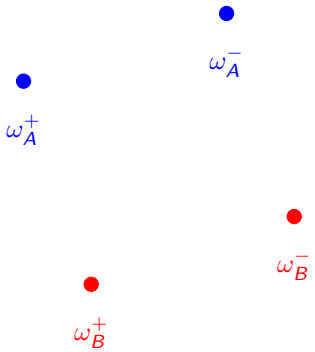
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- Starting point: $\widetilde{\mathcal{G}}^{(2e)}_{AB}\left(\omega^{+}_{A},\omega^{+}_{B}\,|\,\omega^{-}_{A},\omega^{-}_{B}\right)$

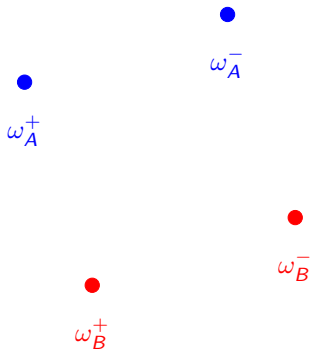
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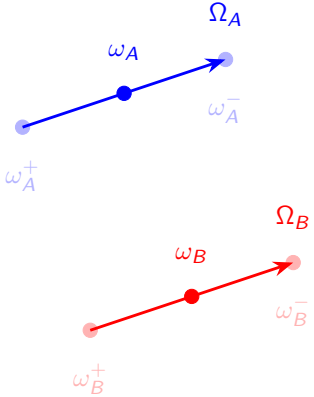
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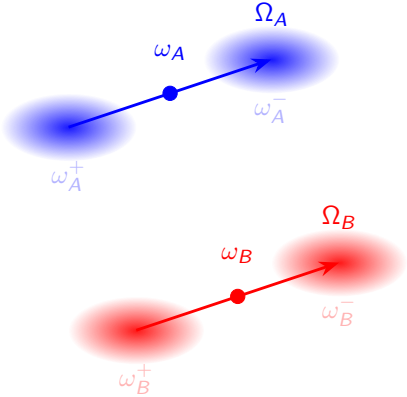
$$\begin{cases} \omega_A = \frac{1}{2}\left(\omega_A^+ + \omega_A^-\right) \\ \omega_B = \frac{1}{2}\left(\omega_B^+ + \omega_B^-\right) \end{cases} \quad \text{and} \quad \begin{cases} \Omega_A = \omega_A^+ - \omega_A^- \\ \Omega_B = \omega_B^+ - \omega_B^- \end{cases}$$



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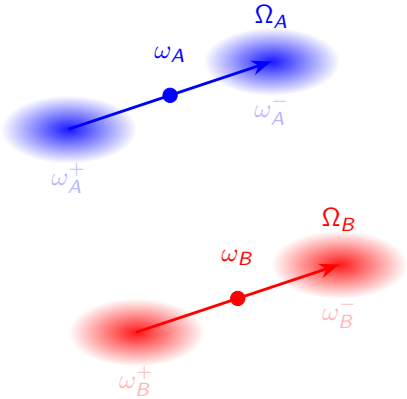


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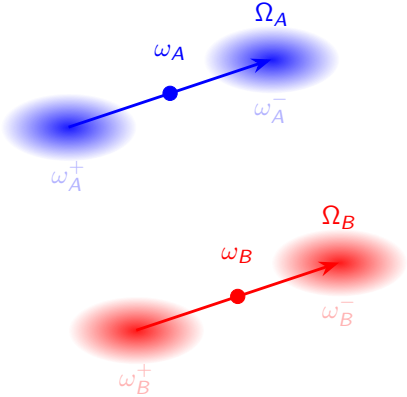
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- Center-of-mass and relative particle for Ω-variables

$$\Omega_\Sigma = \Omega_A + \Omega_B \quad \text{and} \quad \Omega_\Delta = \frac{1}{2}\left(\Omega_A - \Omega_B\right)$$



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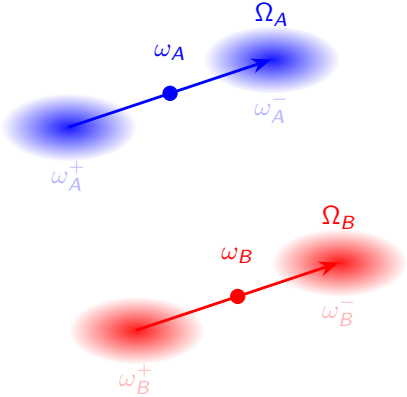
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- Center-of-mass and relative particle for Ω-variables

$$\Omega_\Sigma = \Omega_A + \Omega_B \quad \text{and} \quad \Omega_\Delta = \frac{1}{2}\left(\Omega_A - \Omega_B\right)$$

- New representation: $\widetilde{\mathcal{G}}^{(2e)}_{AB}\left(\omega_A, \omega_B \,|\, \Omega_\Sigma, \Omega_\Delta\right)$



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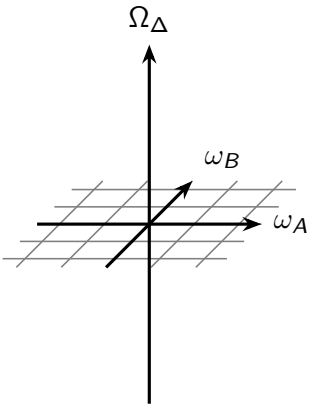
$$\widetilde{\mathcal{G}}^{(2e)}_{AB}\left(\omega_A, \omega_B \,|\, \Omega_\Sigma, \Omega_\Delta\right) \mapsto \widetilde{\mathcal{G}}^{(2e)}_{AB,n}\left(\omega_A, \omega_B \,|\, \Omega_\Delta\right)$$

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- 3D representation: $(\omega_A, \omega_B, \Omega_\Delta)$ space

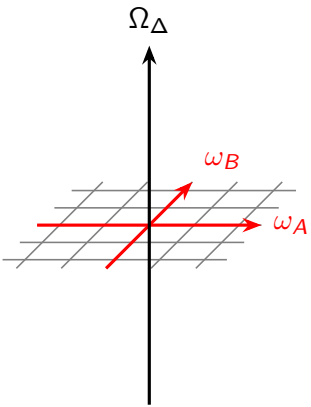


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- 3D representation: $(\omega_A, \omega_B, \Omega_\Delta)$ space
  - (ω_A, ω_B): classical coherence (diagonal plane)

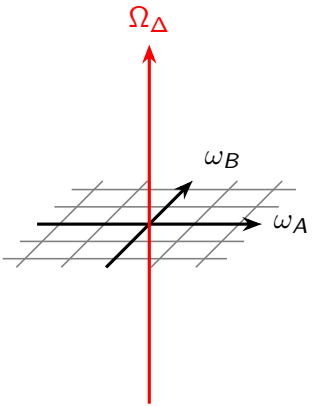


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  - Ω_Δ: quantum coherence (off-diagonal line)



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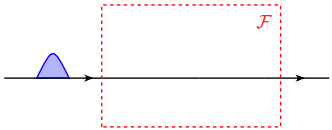
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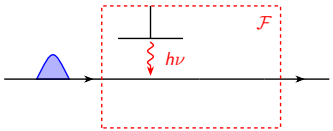
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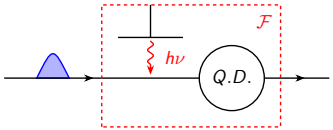
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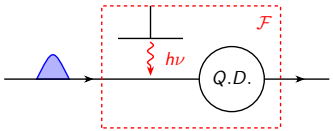
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  - radiation coupler: $|\omega\rangle \to \sum_{n\in\mathbb{Z}} p_n\,|\omega + 2\pi n\nu\rangle$

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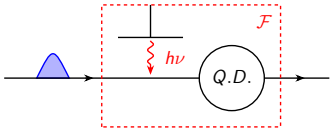
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- Recombine virtual processes with different energies

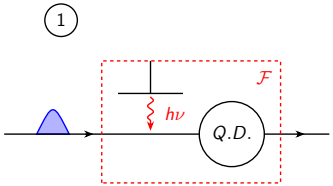
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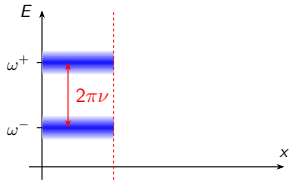




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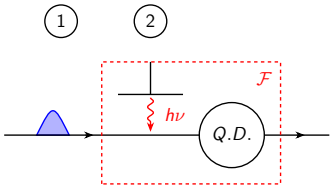
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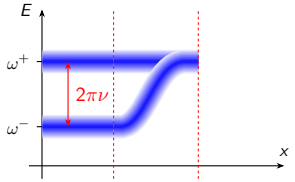




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  - quantum dot to select a single current frequency
- Recombine virtual processes with different energies
  - choose ν so that $\omega^+ = \omega^- + 2\pi\nu$

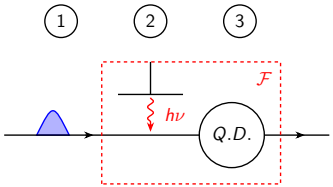
2.1. From the source to the experimental signal

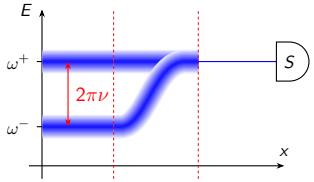




- Photo-assisted filter: $\widetilde{\mathcal{G}}^{(e)}(x\,|\,y) \mapsto$ measurable $\widetilde{\mathcal{G}}^{(e)}(x\,|\,x)$
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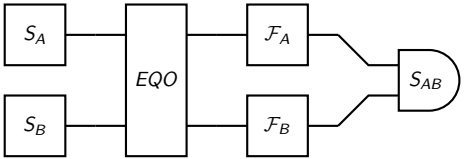
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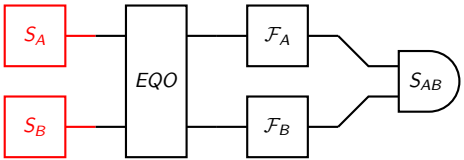
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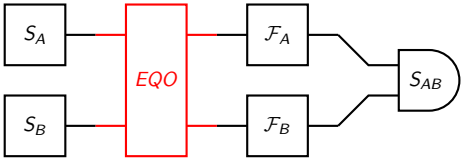
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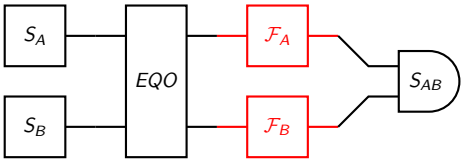
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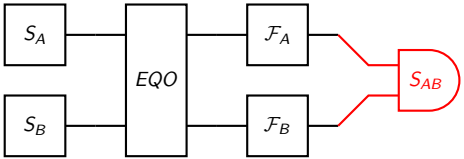
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  - current correlations measurement: reconstruction of $\widetilde{\mathcal{G}}^{(2e)}_{AB}$ in the energy space



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- Gaussian state $\Longrightarrow$ Wick's theorem: $\widetilde{\mathcal{G}}^{(2e)}_{W,AB} = \widetilde{\mathcal{G}}^{(e)}_{A}\,\widetilde{\mathcal{G}}^{(e)}_{B} - \widetilde{\mathcal{G}}^{(e)}_{AB}\,\widetilde{\mathcal{G}}^{(e)}_{BA}$

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- Not a non-interacting / interaction decomposition: interactions contribute to $\widetilde{\mathcal{G}}^{(e)}$

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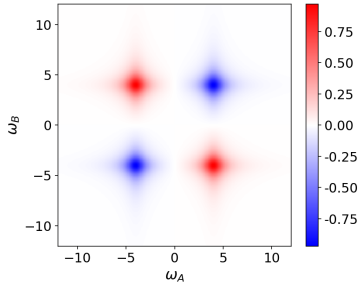
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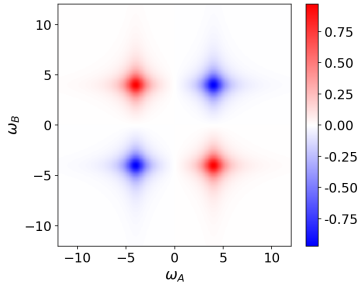
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- Wigner function of connected two-electron coherence: $\Delta_c \mathcal{W}_{AB}^{(2e)} = \mathcal{W}_{AB}^{(2e)} - \mathcal{W}_{A}^{(e)}\, \mathcal{W}_{B}^{(e)}$

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  - no inter-channel tunneling: $\Delta_c \widetilde{\mathcal{G}}^{(2e)}_{AB} = \Delta_W \widetilde{\mathcal{G}}^{(2e)}_{AB}$ $\rightarrow$ see P. Degiovanni's talk!

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$$\begin{aligned}\boldsymbol{Z}_{n,\Omega_\Delta} &= \boldsymbol{X}_{n,\Omega_\Delta} - \left(\boldsymbol{H}_{n,\Omega_\Delta} * \overline{\sin(2\pi nft_\Sigma + \Omega_\Delta t_\Delta - \varphi)\,\mathcal{W}_{AB}^{(e)}\mathcal{W}_{BA}^{(e)}}^{t}\right) \\ &= \boldsymbol{H}_{n,\Omega_\Delta} * \overline{\sin(2\pi nft_\Sigma + \Omega_\Delta t_\Delta - \varphi)\,\Delta_W \mathcal{W}_{AB}^{(2e)}}^{t}\end{aligned}$$

- $\Delta_W \mathcal{W}_{AB}^{(2e)}$ reconstructed with a Bayesian deconvolution algorithm (with physical prior)

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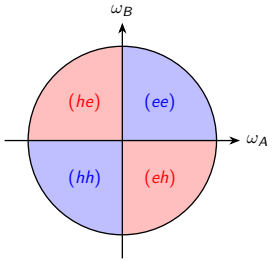
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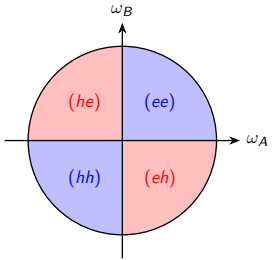
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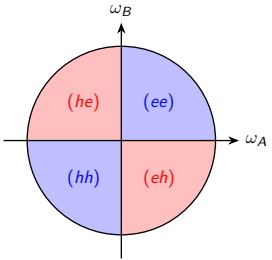
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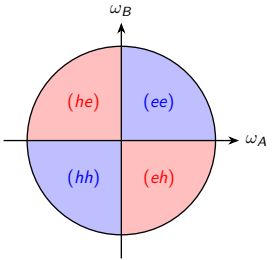
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- Allow to constrain $\Delta_W \widetilde{\mathcal{G}}^{(2e)}_{AB}$ if one chooses a good basis



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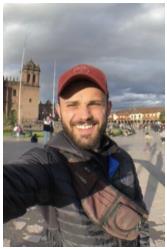
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Thank you for your attention!



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G. Ménard
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Cauchy-Schwarz constraints

Cauchy-Schwarz constraints

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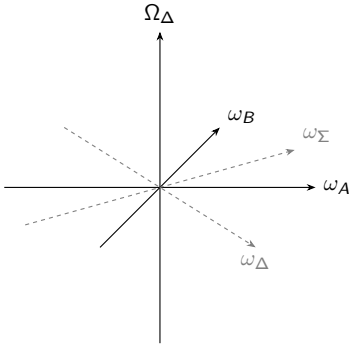
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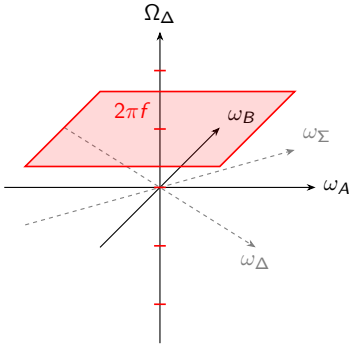
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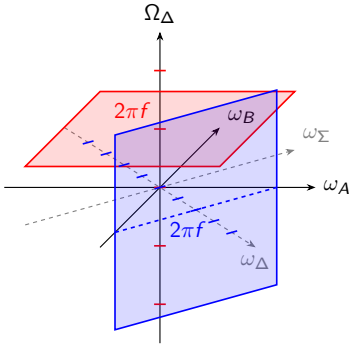
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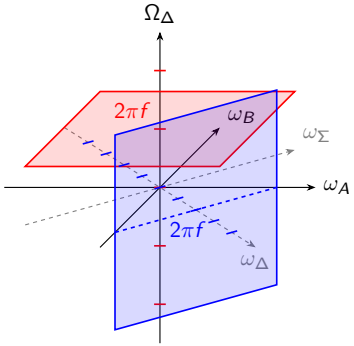


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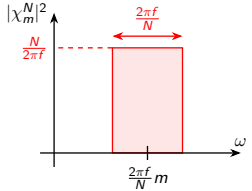
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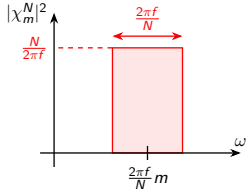
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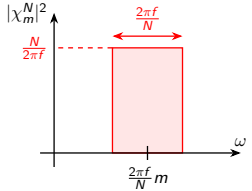
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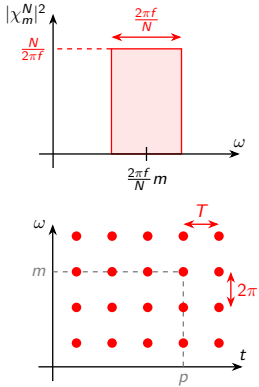
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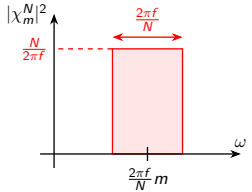
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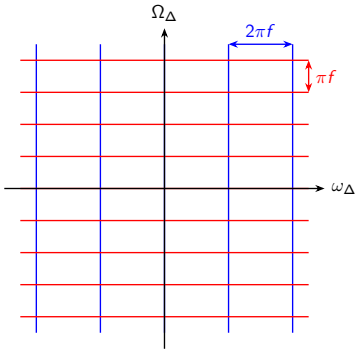
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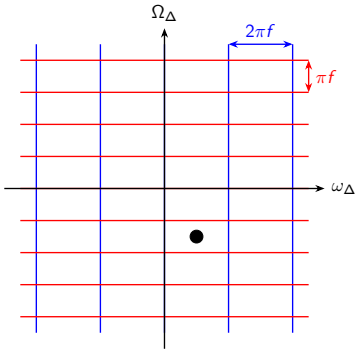
Cauchy-Schwarz constraints

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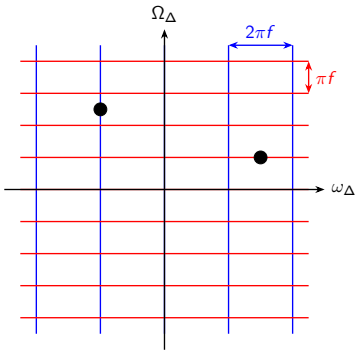
Cauchy-Schwarz constraints

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  - C.-S. l.h.s. out of any singularity plane



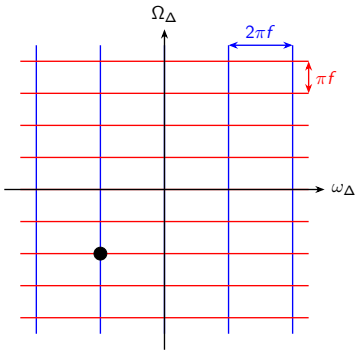
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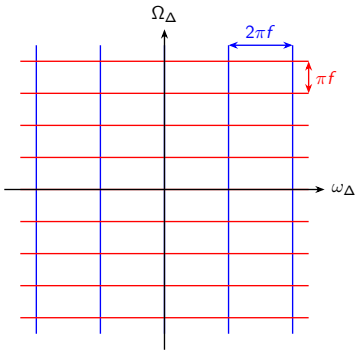
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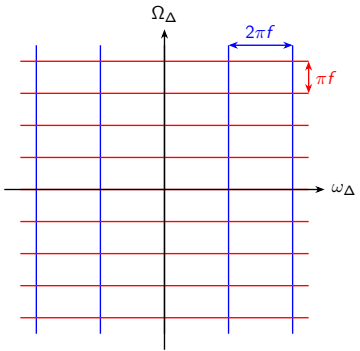
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- Four quadrants $\Rightarrow$ 16 non-independent constraints



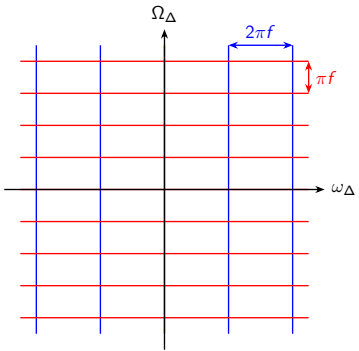
Cauchy-Schwarz constraints

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  - Accessible information is basis-dependent



Cauchy-Schwarz constraints

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  - C.-S. l.h.s. out of any singularity plane
  - C.-S. l.h.s. on exactly one singularity plane (two-cases)
  - C.-S. l.h.s. on the intersection of two singularity planes
- Four quadrants $\Rightarrow$ 16 non-independent constraints
- Finite set of constraints in an infinite basis
  - Accessible information is basis-dependent
  - Link information from different basis: work in progress

