Probing collision-induced electronic entanglement in ballistic conductors

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Latvijas Universitātes Fizikas nodaļa, Riga

Introduction

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- Mesoscopic conductors in the quantum Hall regime

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- Yesterday: P. Degiovanni's talk on two-electron photo-assisted tomography protocol

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- Theoretical collision model between two electronic wavepackets
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  - coherent collision: no energy leaks to the environment
  - high-energy electrons: no perturbation of the Fermi sea

Outline

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1.1. First-order electronic coherence function

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$$\mathcal{G}^{(1e)}_{\hat{\rho}}(t^+ \mid t^-) = \left\langle \hat{\psi}^\dagger(t^-)\, \hat{\psi}(t^+) \right\rangle_{\hat{\rho}} = Tr\left(\hat{\rho}\, \hat{\psi}^\dagger(t^-)\, \hat{\psi}(t^+)\right)$$

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- Useful to access the dynamics of the system

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- Nature of the electronic-excitations (content in energy)

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$$\mathcal{W}^{(1e)}_{\hat{\rho}}(t\,|\,\omega) = v_F \int_{\mathbb{R}} d\tau\, \mathcal{G}^{(1e)}_{\hat{\rho}}(t\,|\,\tau)\, e^{i\omega\tau} = v_F \int_{\mathbb{R}} \frac{d\Omega}{2\pi}\, \widetilde{\mathcal{G}}^{(1e)}_{\hat{\rho}}(\Omega\,|\,\omega)\, e^{-i\Omega t}$$

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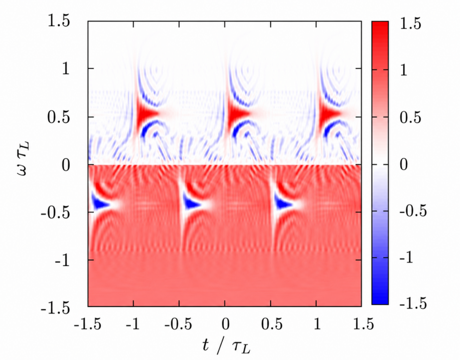
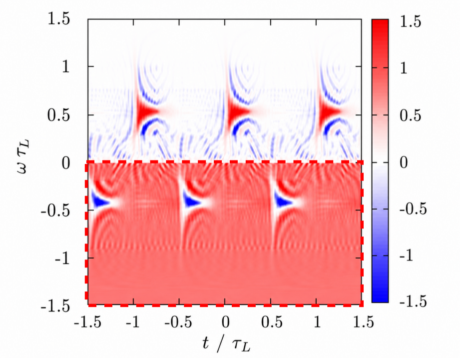


1.1. First-order electronic coherence function

- Wigner function of a periodic source
  - Fermi sea (population $n = 1$) for $\omega\,\tau_L < 0$



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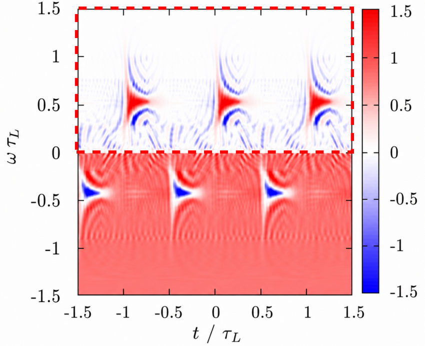

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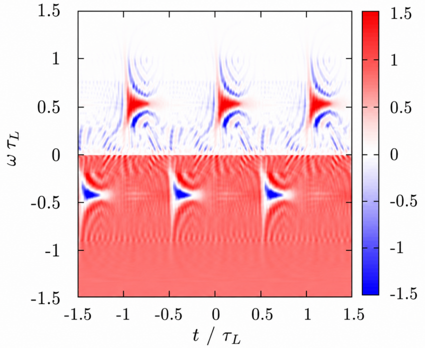

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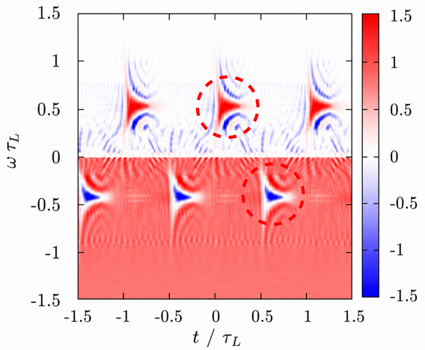

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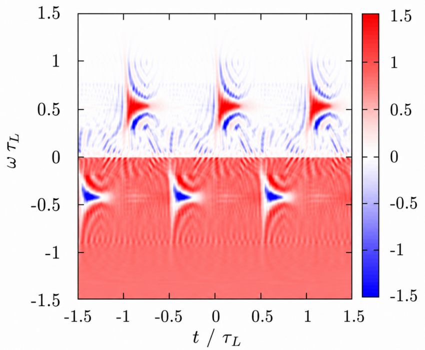

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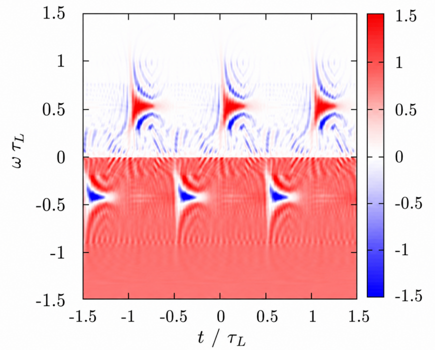

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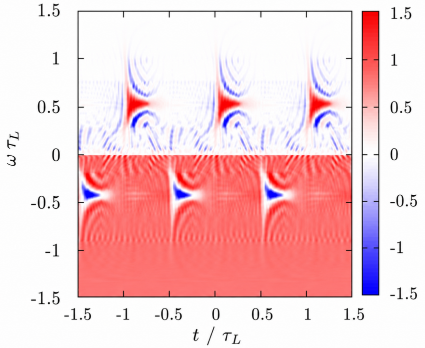

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  - find a 3D representation: better visualization in the conjugate space

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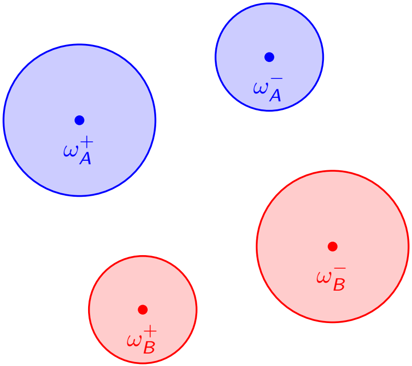

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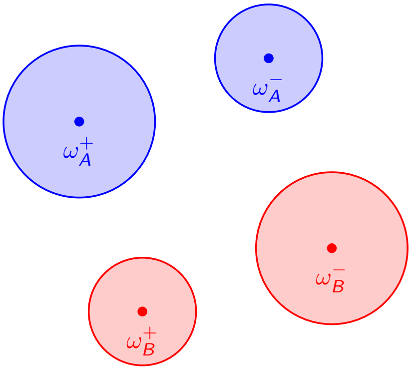

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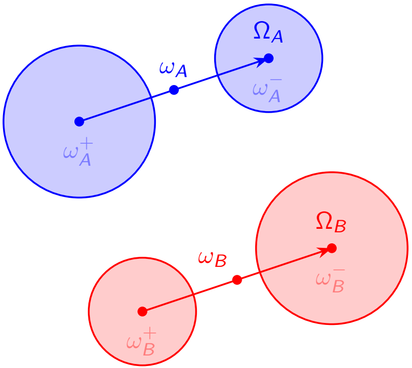

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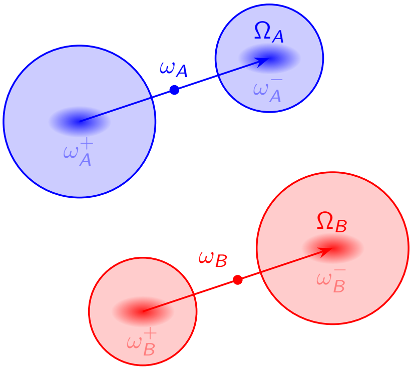

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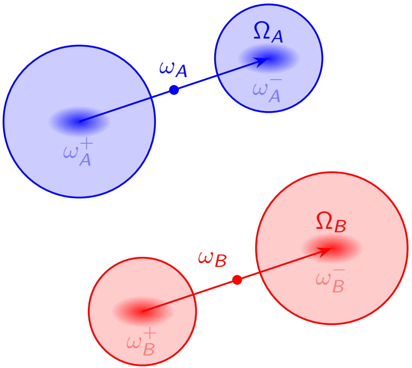
- Center of mass and relative particle for Ω-variables



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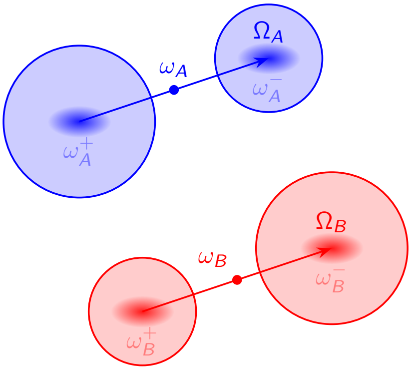

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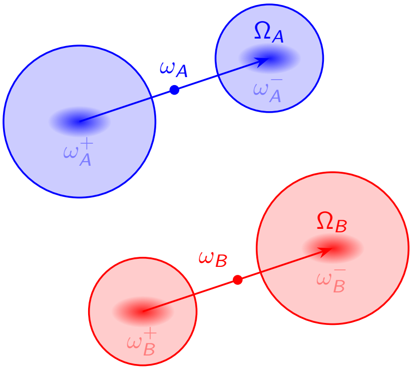

- Center of mass and relative particle for Ω-variables

$$\Omega_\Sigma = \Omega_A + \Omega_B \quad \text{and} \quad \Omega_\Delta = \frac{1}{2}(\Omega_A - \Omega_B)$$

- New representation: $\widetilde{\mathcal{G}}^{(2e)}_{\hat{\rho}_{AB}}(\omega_A,\omega_B\,|\,\Omega_\Sigma,\Omega_\Delta)$

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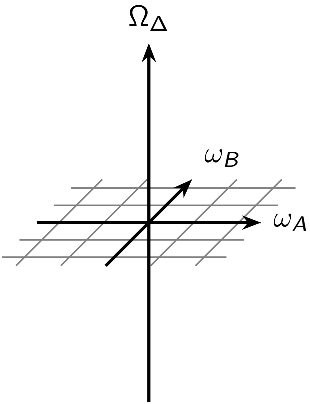

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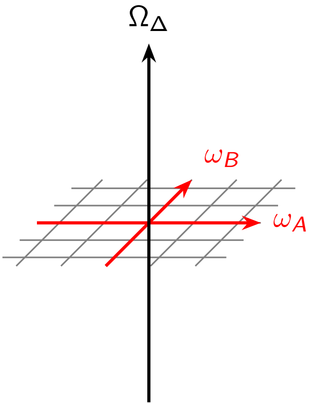

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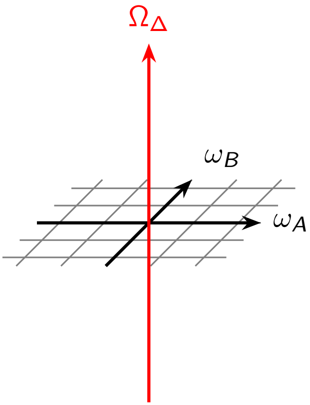

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  - study in the $(\omega_A, \omega_B, \Omega_\Delta)$ 3D Fourier subspace: classical plane + quantum line

Outline

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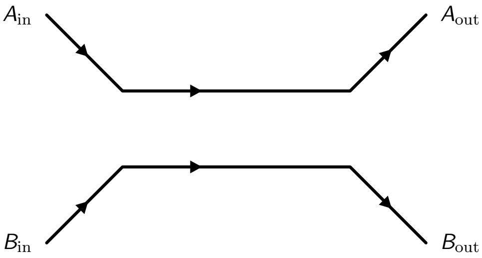

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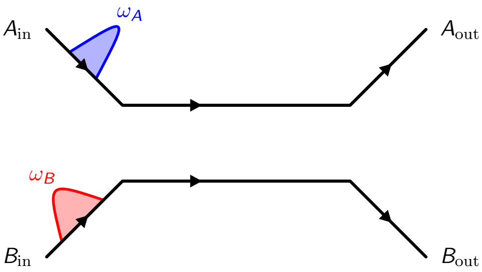

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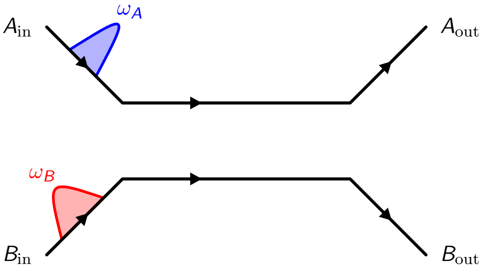

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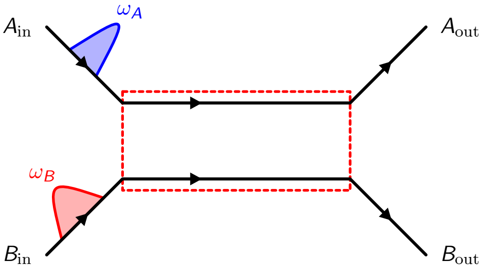

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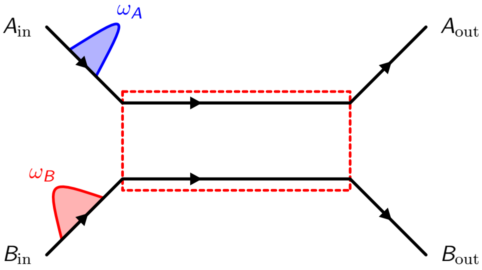

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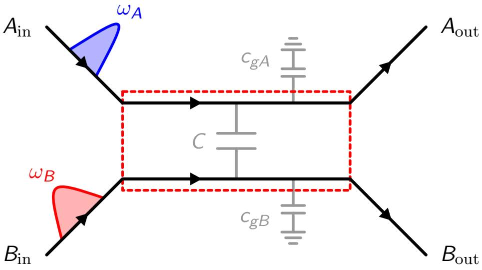

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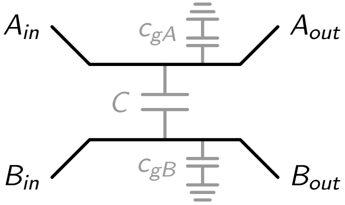

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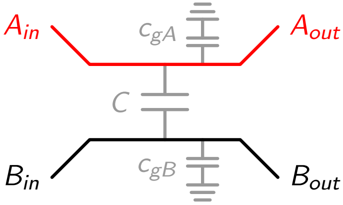

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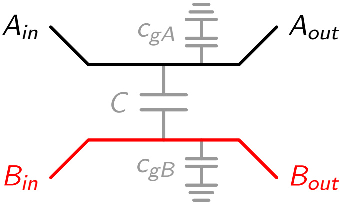

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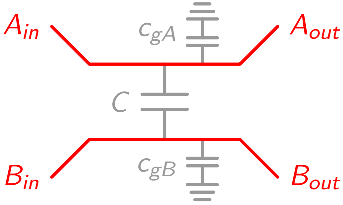

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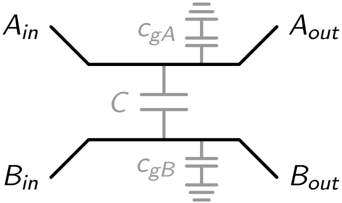

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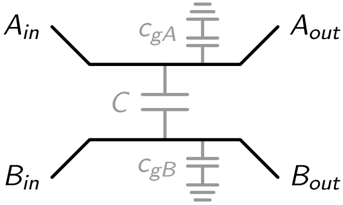

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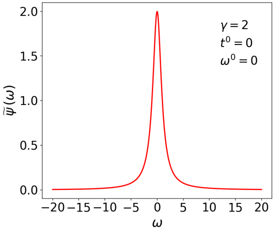

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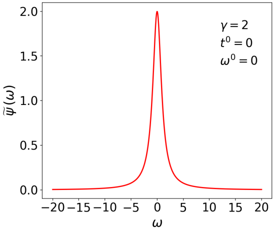

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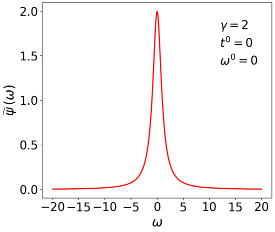

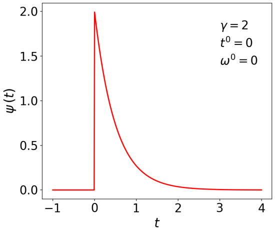

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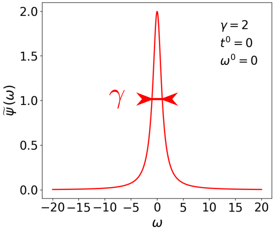

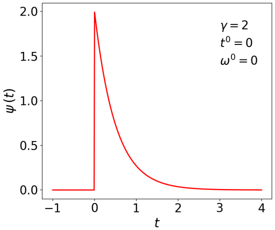

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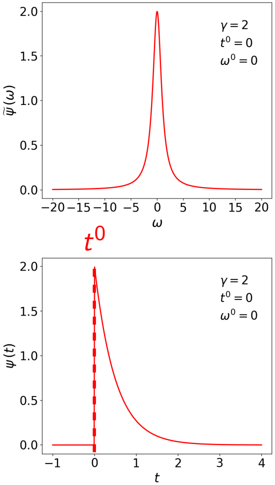

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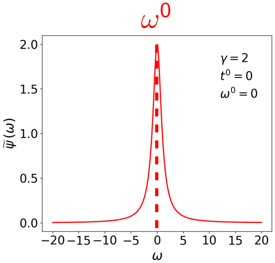

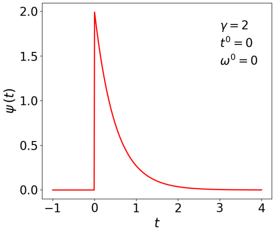

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- Scattering matrix: $S_{\omega_A^0,\omega_B^0}(\omega_A, \omega_B) = \left\langle \widetilde{\phi}(\omega_A, \omega_B) \right| \hat{K} \left| \widetilde{\psi}(\omega_A^0, \omega_B^0) \right\rangle$

2.2. Collision of two Landau wavepackets

$$\widetilde{\mathcal{G}}^{(2e)}_{\hat{\rho}_{AB}}(\omega_A^+, \omega_B^+ \,|\, \omega_A^-, \omega_B^-) = S_{\omega_A^0,\omega_B^0}(\omega_A^+, \omega_B^+)\, S_{\omega_A^0,\omega_B^0}(\omega_A^-, \omega_B^-)^*$$

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  - plane waves: detection at a given (punctual) energy

$$\widetilde{\mathcal{G}}^{(2e)}_{\hat{\rho}_{AB}}(\omega_A^+,\omega_B^+\,|\,\omega_A^-,\omega_B^-) = \int_{\mathbb{R}^4} d^4\mathbf{t}\; e^{i\boldsymbol{\omega}^+\cdot\mathbf{t}^+} K(\boldsymbol{t}^+|\,\boldsymbol{t}^0)\psi(\boldsymbol{t}^0)\; e^{-i\boldsymbol{\omega}^-\cdot\mathbf{t}^-} K(\boldsymbol{t}^-|\,\boldsymbol{t}^0)^*\psi(\boldsymbol{t}^0)^*$$

2.2. Collision of two Landau wavepackets

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- Diagonal coherence (energy probability distribution)

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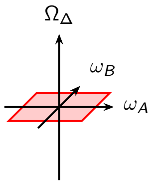

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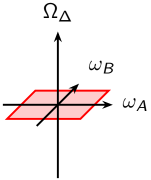
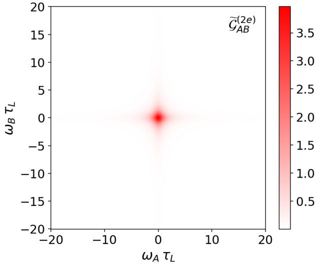
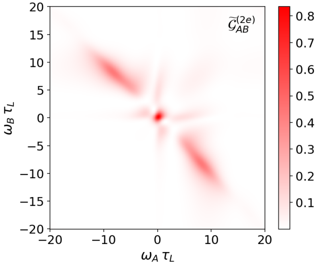


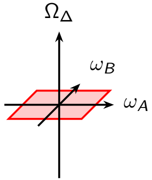


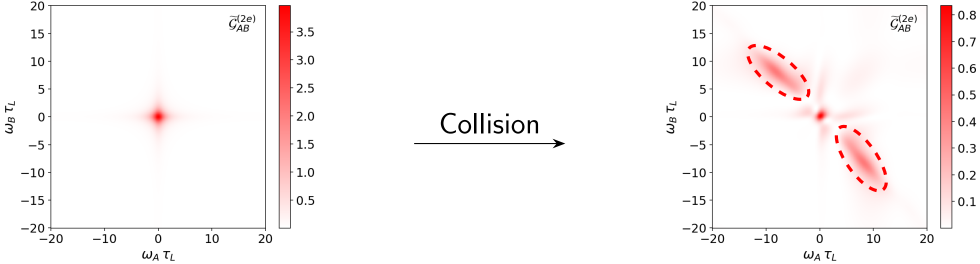
Collision →



2.2. Collision of two Landau wavepackets

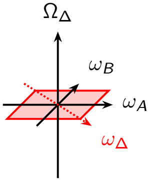
- Diagonal coherence (energy probability distribution)
  - Lateral lobes: energy transfers $A \to B$ and $B \to A$

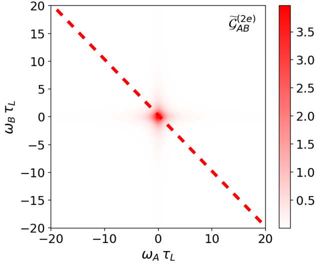




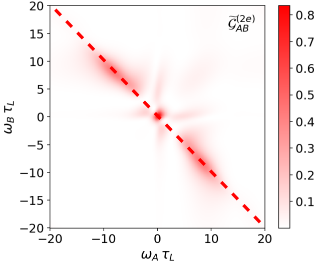
2.2. Collision of two Landau wavepackets

- Diagonal coherence (energy probability distribution)
  - Lateral lobes: energy transfers $A \to B$ and $B \to A$
  - spread along $\omega_\Delta = \frac{1}{2}(\omega_A - \omega_B)$: energy conservation





Collision →



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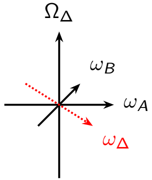

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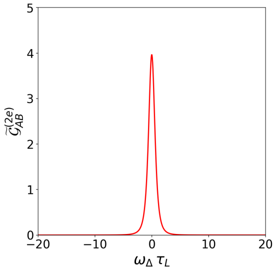
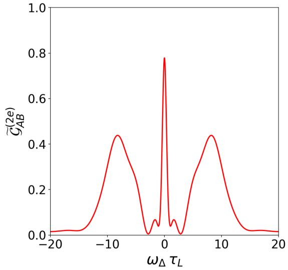


Collision →



2.2. Collision of two Landau wavepackets

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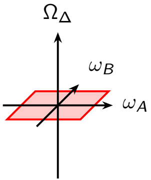
- Evolution with coupling parameter α



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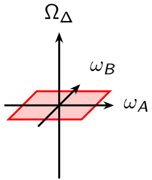
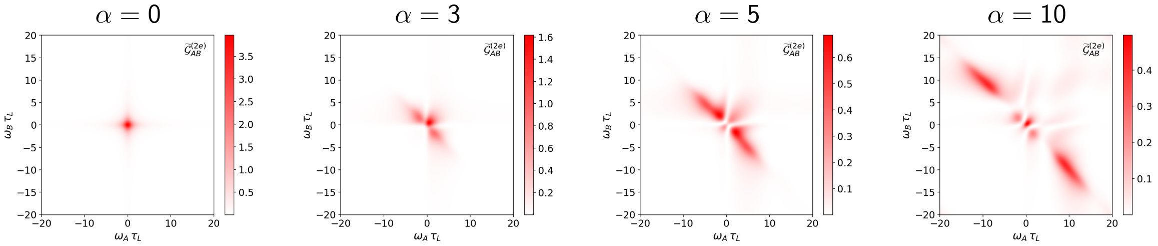
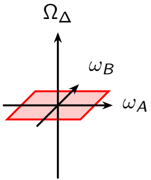
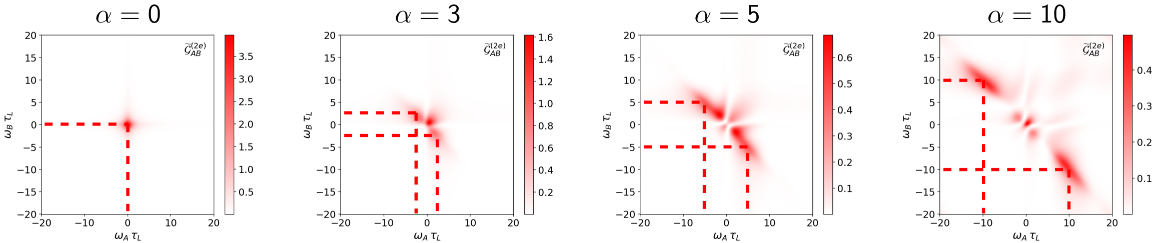




2.2. Collision of two Landau wavepackets

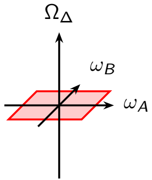
- Evolution with coupling parameter α
  - lobes around $\omega\tau_L = \pm\alpha$: exchanged energy increases with α

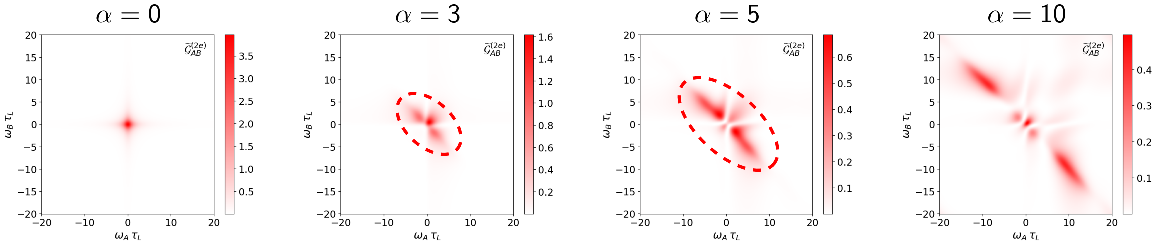




2.2. Collision of two Landau wavepackets

- Evolution with coupling parameter α
  - lobes around $\omega\tau_L = \pm\alpha$: exchanged energy increases with α
  - α too weak $\implies$ lobes not clearly resolved





2.2. Collision of two Landau wavepackets

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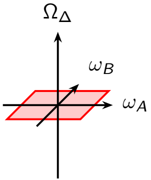
- Influence of the injection time offset $t_\Delta^0 = t_A^0 - t_B^0$



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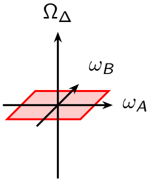
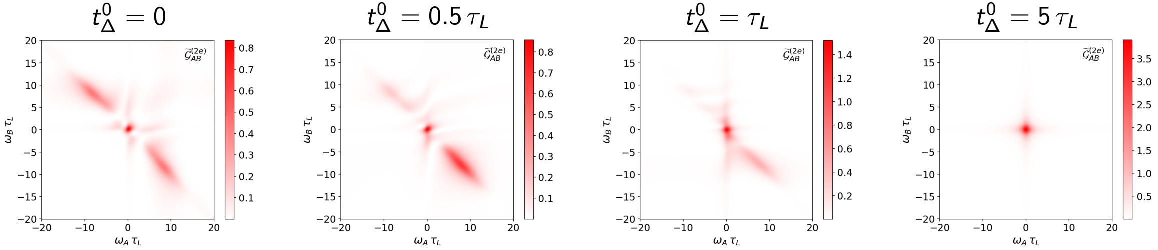
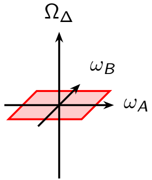
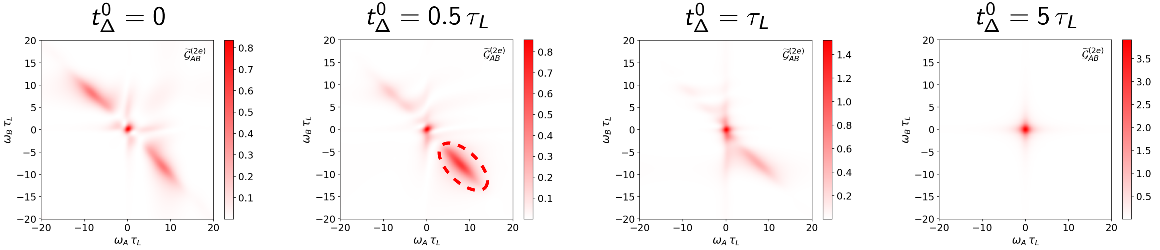




2.2. Collision of two Landau wavepackets

- Influence of the injection time offset $t_\Delta^0 = t_A^0 - t_B^0$
  - $0 < t_\Delta^0 < \tau_L \implies$ one transfer preferred over the other





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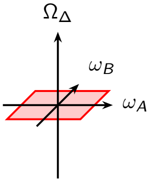
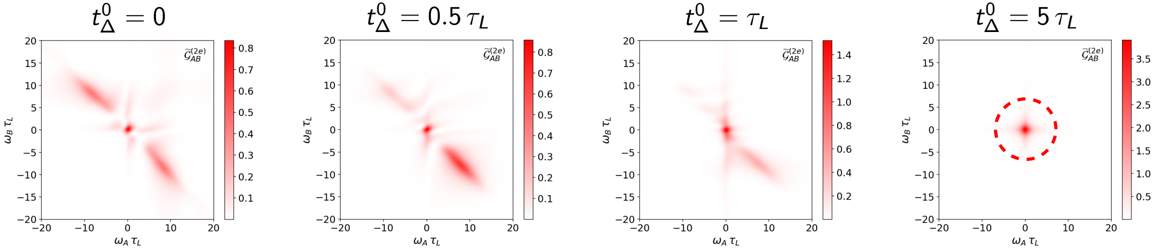
- Influence of the injection time offset $t_\Delta^0 = t_A^0 - t_B^0$
  - $0 < t_\Delta^0 < \tau_L \implies$ one transfer preferred over the other
  - $t_\Delta^0 > \tau_L$: electrons don't see each other ($\widetilde{\mathcal{G}}^{(2e)}_{\hat\rho_{AB},out} = \widetilde{\mathcal{G}}^{(2e)}_{\hat\rho_{AB},in}$)





2.2. Collision of two Landau wavepackets

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- Snowplough effect: the late electron takes energy from the first electron

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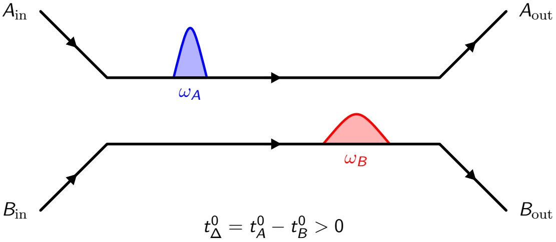
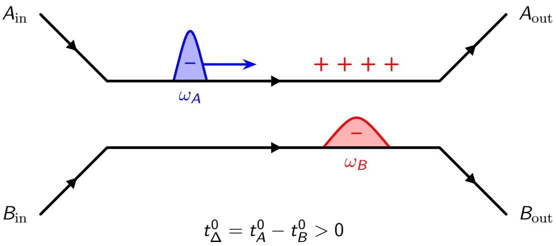


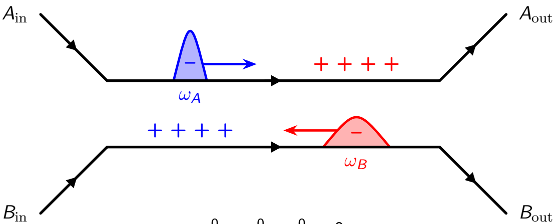
2.2. Collision of two Landau wavepackets

- Snowplough effect: the late electron takes energy from the first electron
  - first electron polarizes other channel $\Longrightarrow$ accelerates second electron



2.2. Collision of two Landau wavepackets

- Snowplough effect: the late electron takes energy from the first electron
  - first electron polarizes other channel $\implies$ accelerates second electron
  - reverse process: second electron slows first electron down

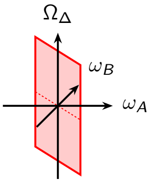


$$t_\Delta^0 = t_A^0 - t_B^0 > 0$$

2.2. Collision of two Landau wavepackets

2.2. Collision of two Landau wavepackets

- Off-diagonal coherence (quantum component)



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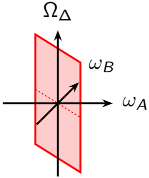
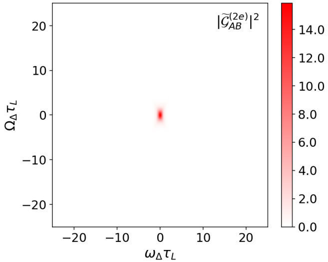
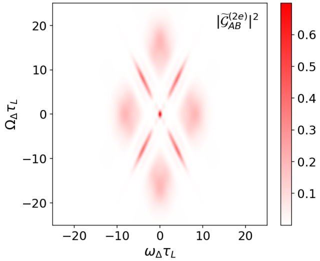




Collision



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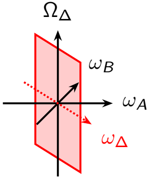
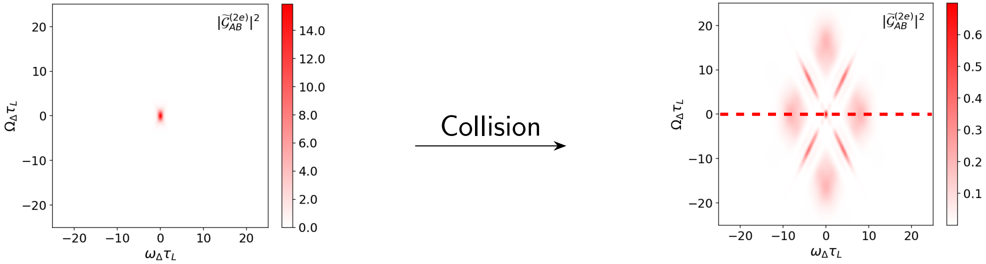
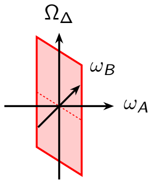
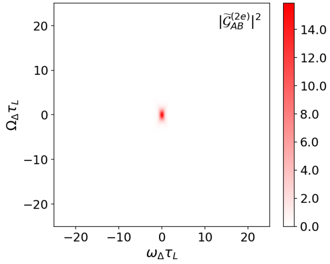


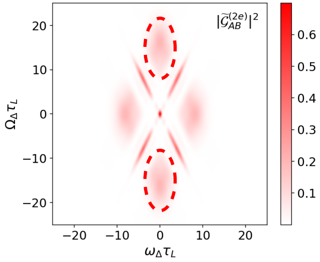


2.2. Collision of two Landau wavepackets

- Off-diagonal coherence (quantum component)
  - vertical lobes: interferences between lateral lobes

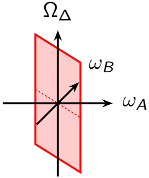


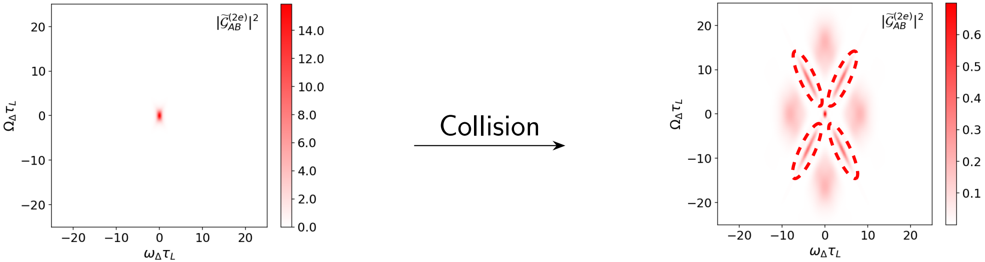




2.2. Collision of two Landau wavepackets

- Off-diagonal coherence (quantum component)
  - vertical lobes: interferences between lateral lobes
  - fringes: interferences central structure / lateral lobes





2.2. Collision of two Landau wavepackets

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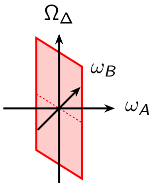
- Evolution with coupling parameter α



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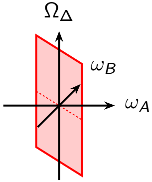
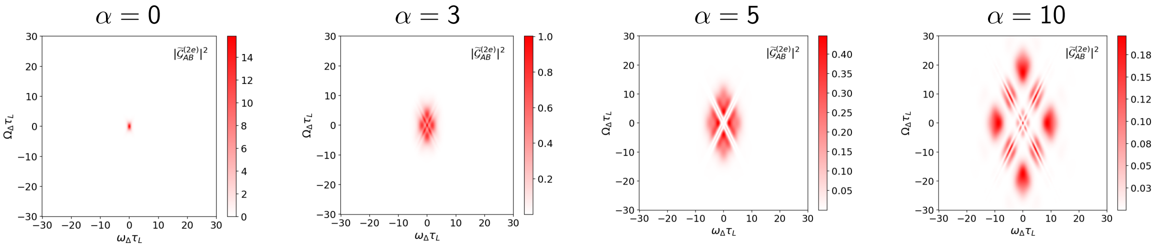
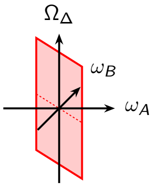
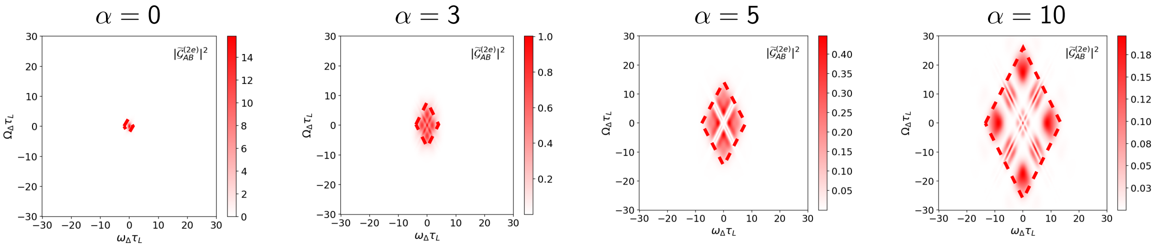




2.2. Collision of two Landau wavepackets

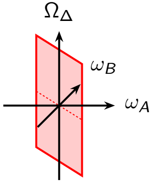
- Evolution with coupling parameter α
  - diamond shape growing with α: Cauchy-Schwarz inequality

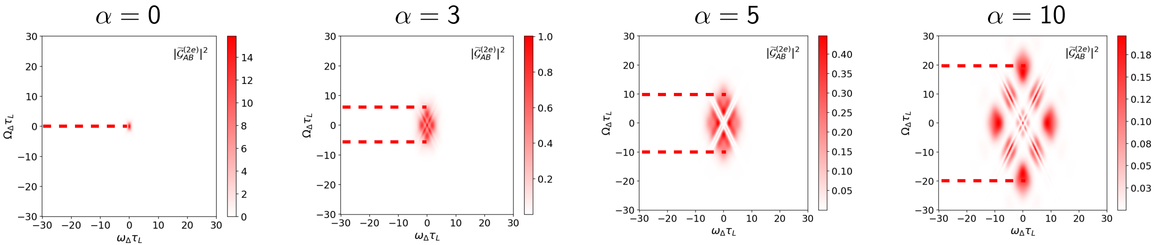




2.2. Collision of two Landau wavepackets

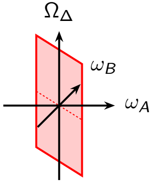
- Evolution with coupling parameter α
  - diamond shape growing with α: Cauchy-Schwarz inequality
  - vertical lobes at $\Omega_\Delta \tau_L = \pm 2\alpha$: quantum component $\propto$ coupling

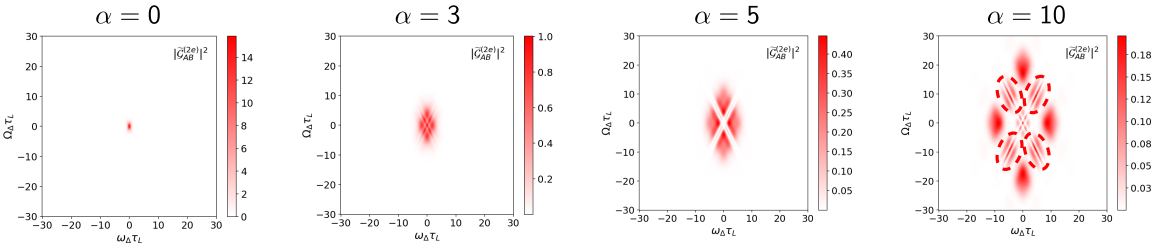




2.2. Collision of two Landau wavepackets

- Evolution with coupling parameter α
  - diamond shape growing with α: Cauchy-Schwarz inequality
  - vertical lobes at $\Omega_\Delta \tau_L = \pm 2\alpha$: quantum component $\propto$ coupling
  - more interference fringes for high α: more exchange processes





2.2. Collision of two Landau wavepackets

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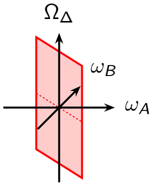
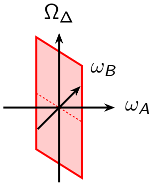
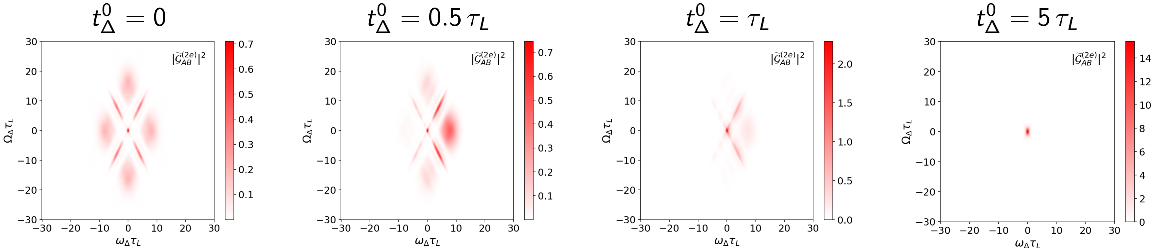
- Influence of the injection time offset t_{Δ}^{0}



2.2. Collision of two Landau wavepackets

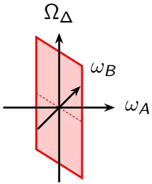
- Influence of the injection time offset t^0_Δ

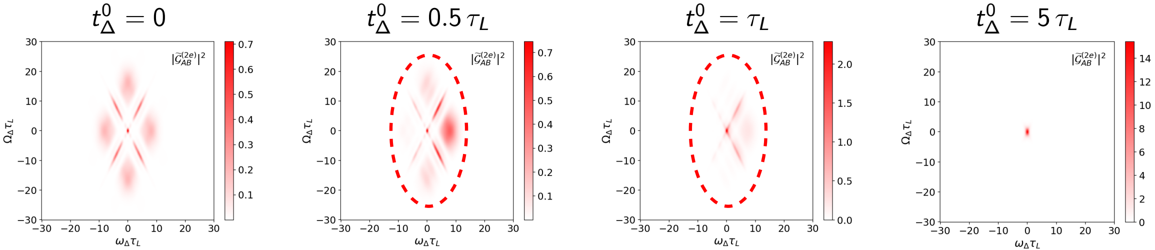




2.2. Collision of two Landau wavepackets

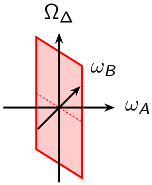
- Influence of the injection time offset t_Δ^0
  - snowplough effect $\Longrightarrow$ fading of corresponding lobes and fringes

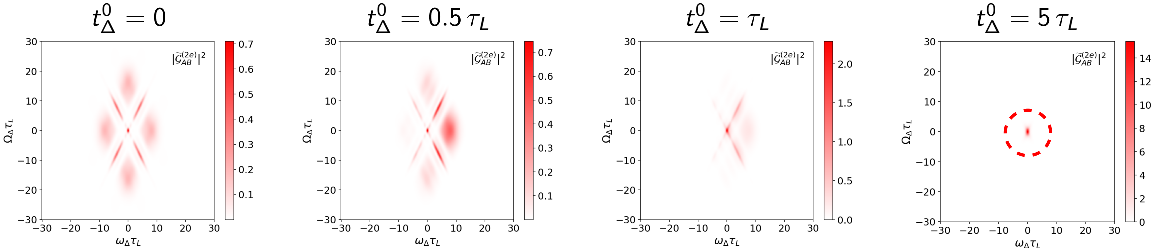




2.2. Collision of two Landau wavepackets

- Influence of the injection time offset t_Δ^0
  - snowplough effect $\Longrightarrow$ fading of corresponding lobes and fringes
  - $t_\Delta^0 \gg \tau_L$: coherence collapses to its input value ($\Longleftrightarrow$ $\alpha = 0$)





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  - injection-time offset $\Longrightarrow$ energy transfers become asymmetric

Outline

3.1. Cauchy-Schwarz entanglement witness

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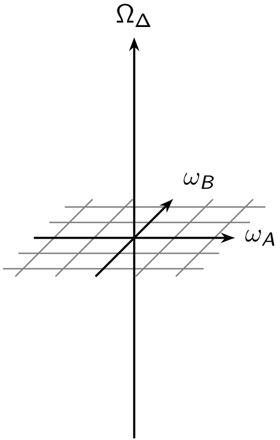

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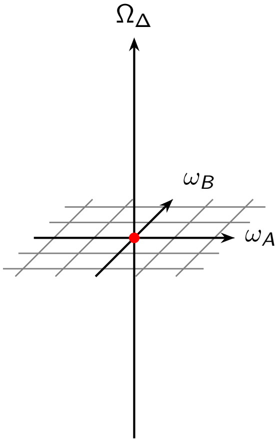

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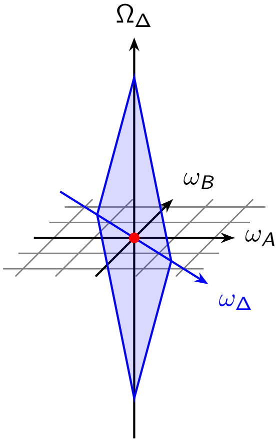

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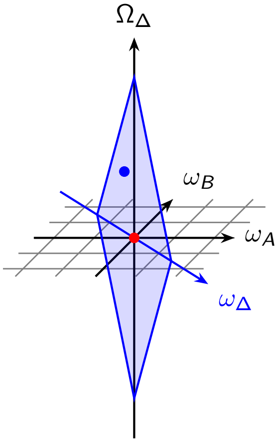

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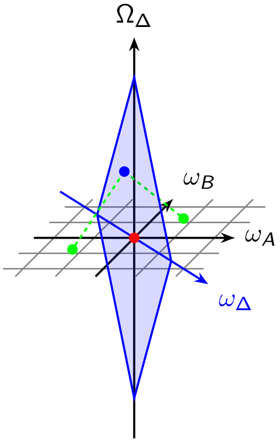

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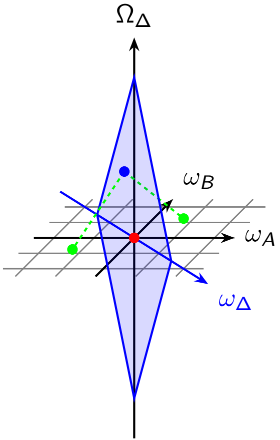
- RHS points outside of the diamond $\implies$ witness violated



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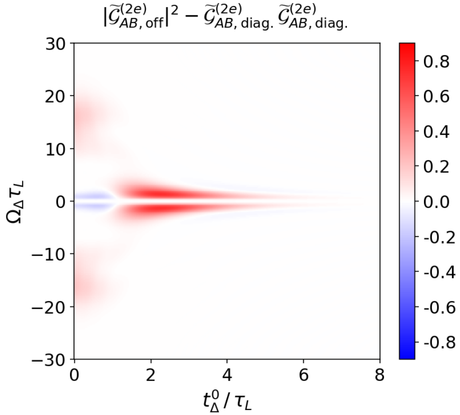

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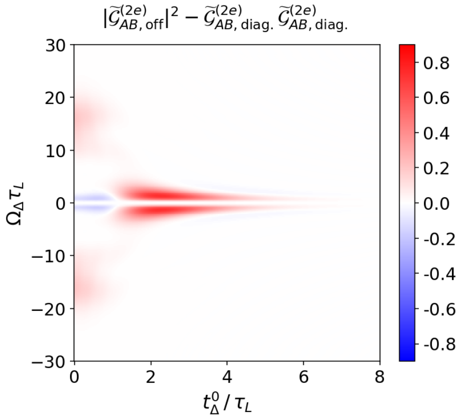
- LHS - RHS: positive $\Longleftrightarrow$ violated
- Two violation regions



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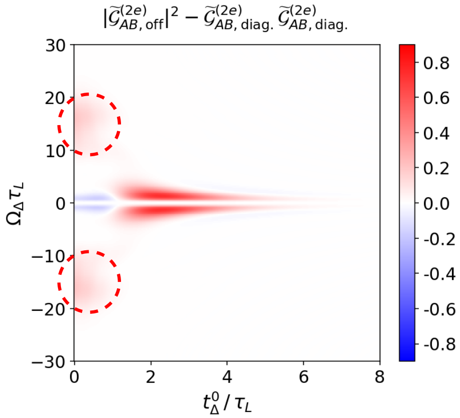

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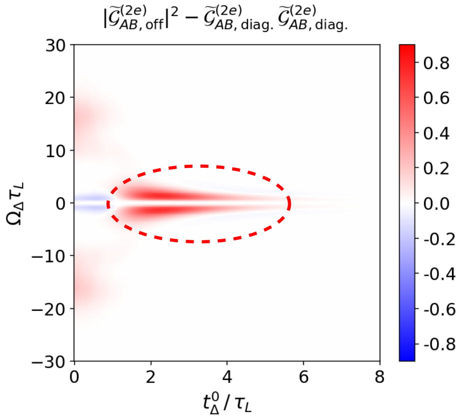

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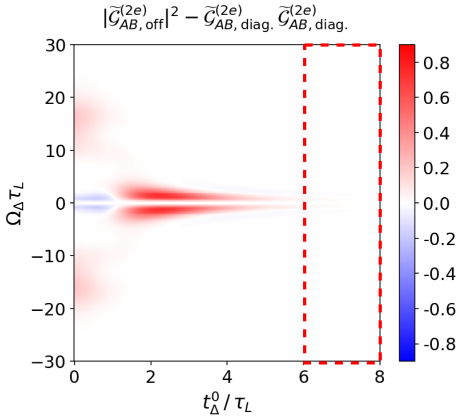

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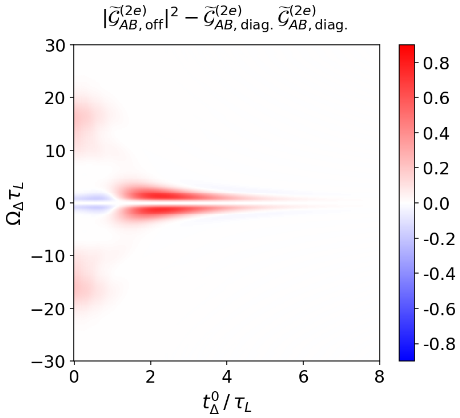
- Entanglement detection even when $t_\Delta^0 > \tau_L$!



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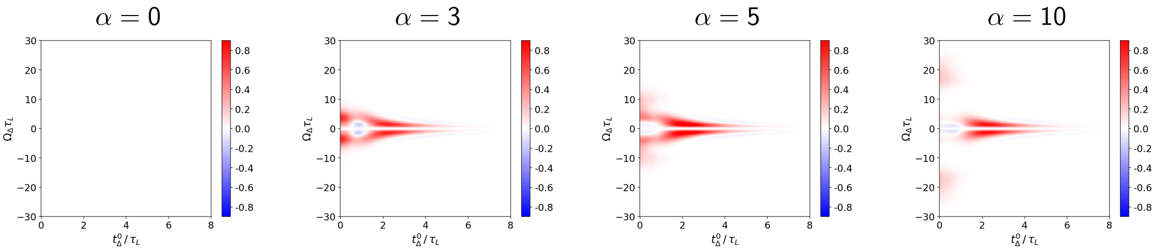

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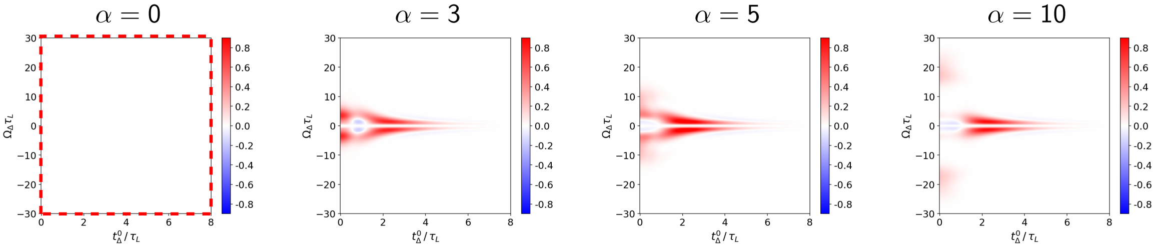

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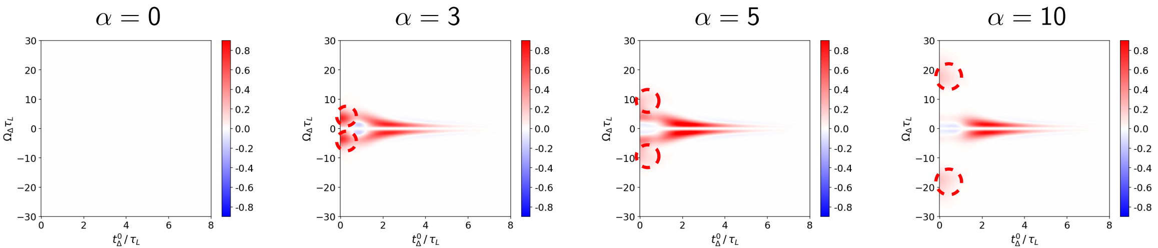

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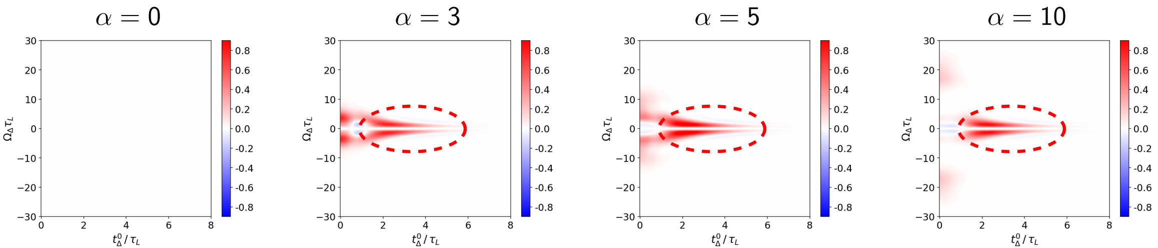
  - nearly-diagonal violation region very resilient: present for all non-zero coupling strengths



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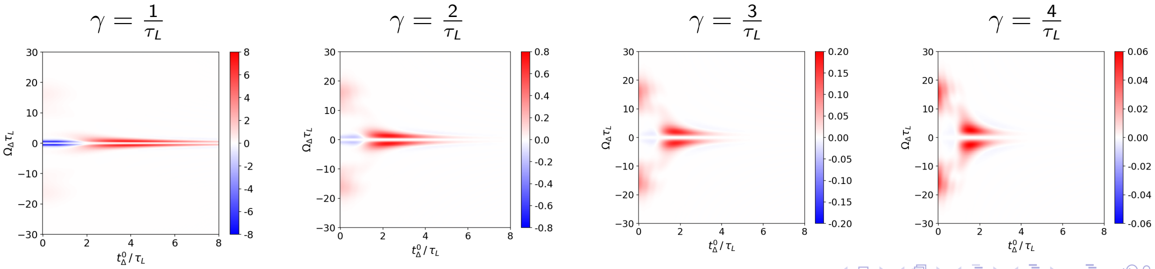

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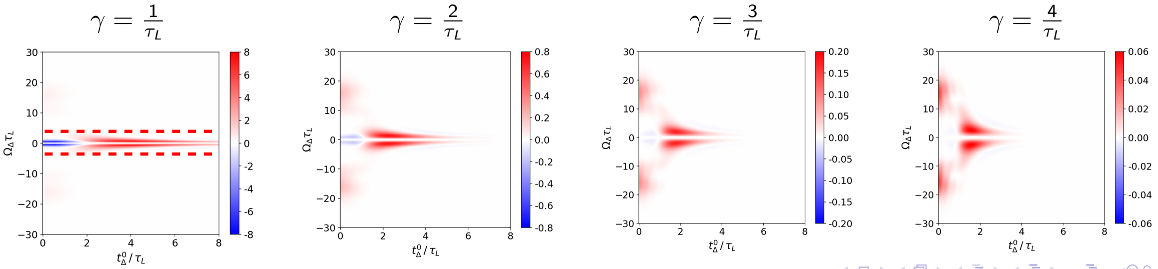

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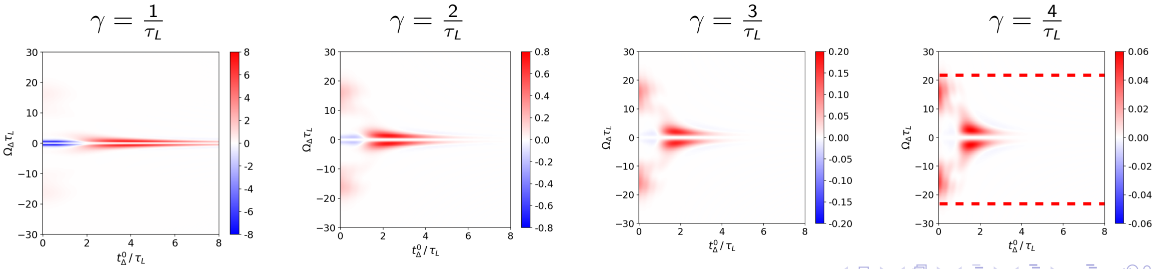

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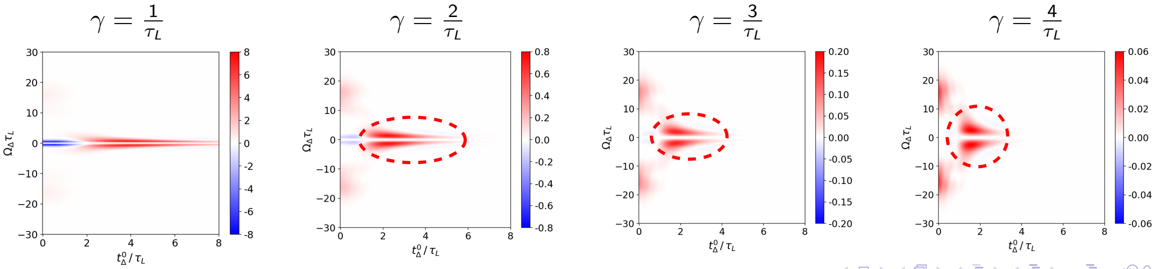
  - increasing γ limits temporal overlap between wavepackets $\Longrightarrow$ signal fades quicker



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- Time-frequency representation: Wigner function of the two-electron state

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  - integrate along the time of the center of mass

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- Time-frequency representation: Wigner function of the two-electron state

$$\mathcal{W}^{(1e)}_{\Delta}(t_\Delta \,|\, \omega_\Delta) = \int_{\mathbb{R}} dt_\Sigma \, \mathcal{W}^{(2e)}_{\hat{\rho}_{AB}}(t_\Sigma, t_\Delta \,|\, \omega_\Sigma, \omega_\Delta)\Big|_{\omega_\Sigma = \omega_A^0 + \omega_B^0}$$

- Quantum correlations $\Longrightarrow$ off-diagonal coherence: quantum variables
  - fix the energy of the center of mass to its initial value
  - integrate along the time of the center of mass
- Wigner function for the relative particle: coherence in the 2D $(t_\Delta, \omega_\Delta)$ plane

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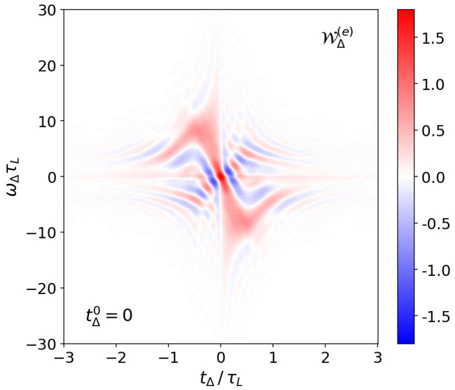

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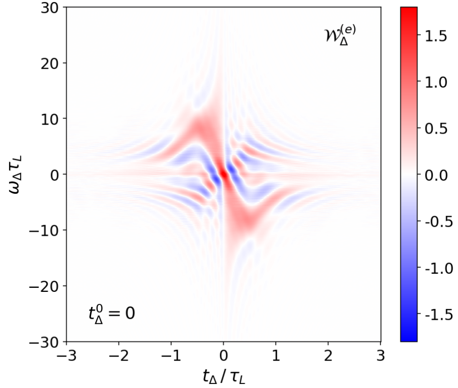

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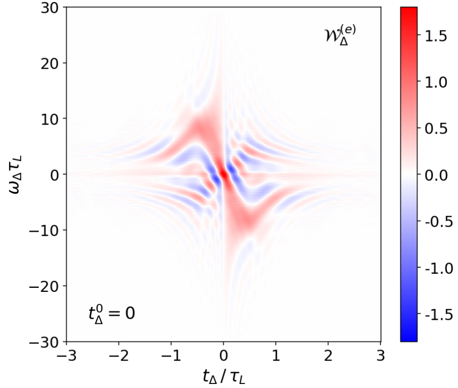

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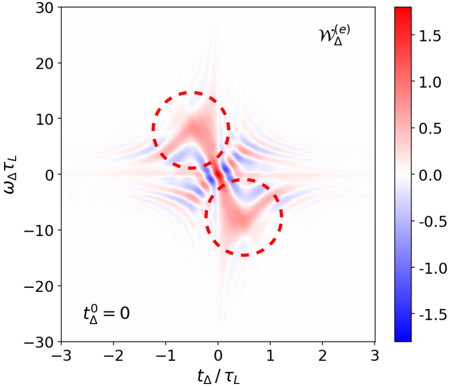

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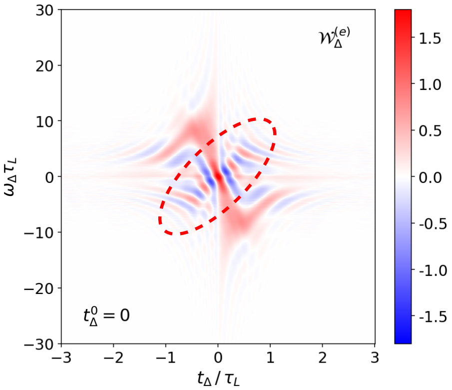

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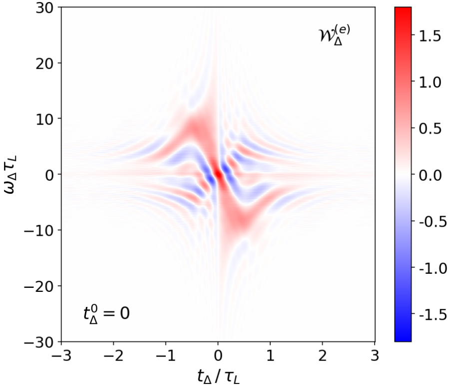

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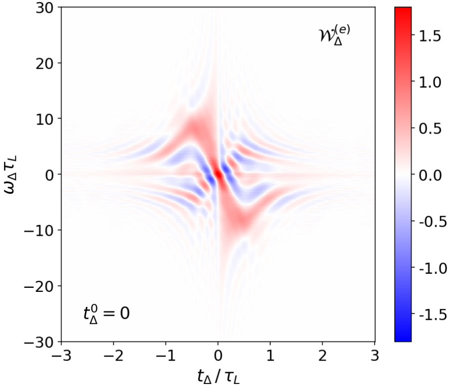
  - negativities $\Longleftrightarrow$ quantum effects
- Wigner representation highlights entanglement!



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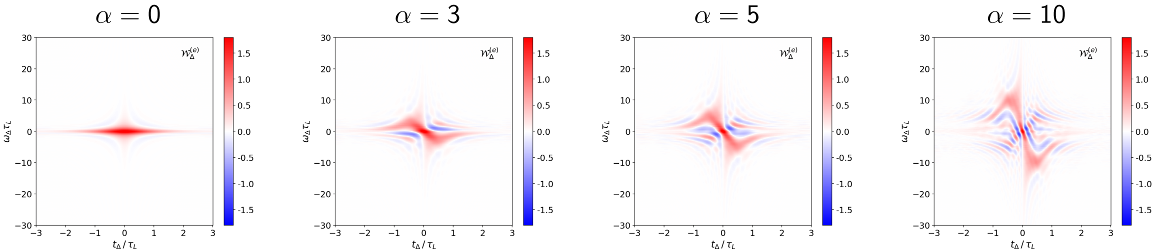

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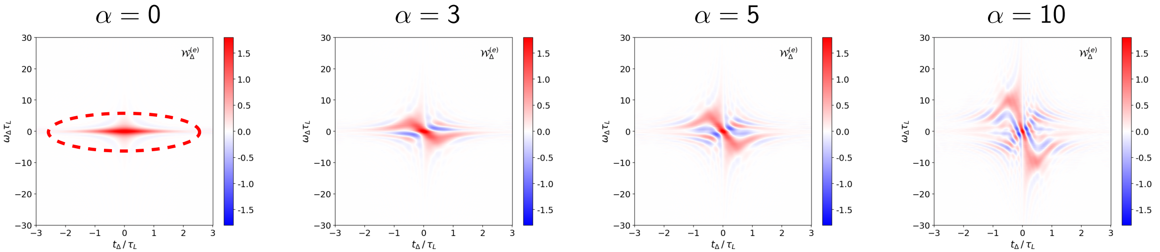
- Evolution with coupling parameter α
  - $\alpha = 0$: no interactions $\Longrightarrow$ diagonal coherence and no negativities



3.2. Signatures of collision-induced entanglement

- Evolution with coupling parameter α
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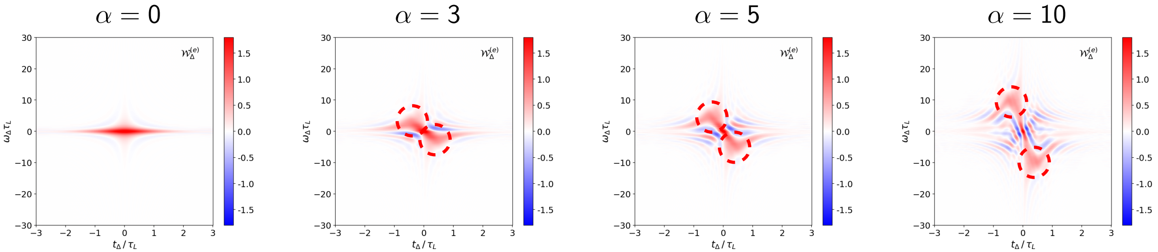
  - Positive lobes around $\omega_\Delta \tau_L = \pm\alpha$: transferred energy increases with α



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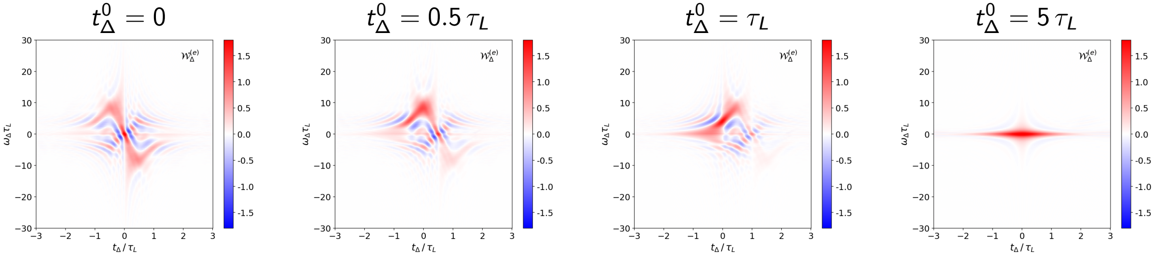

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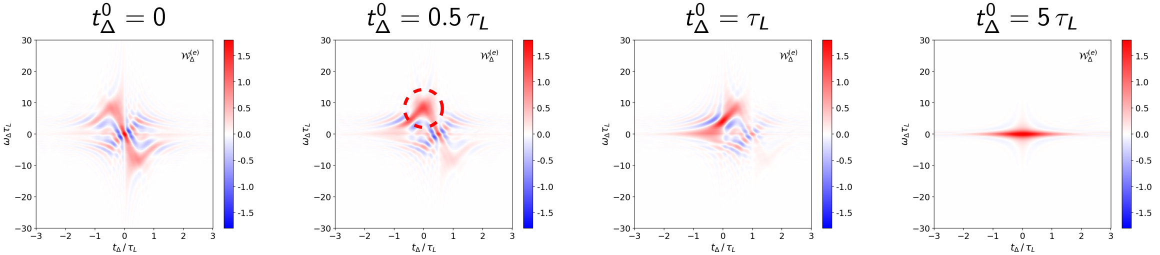

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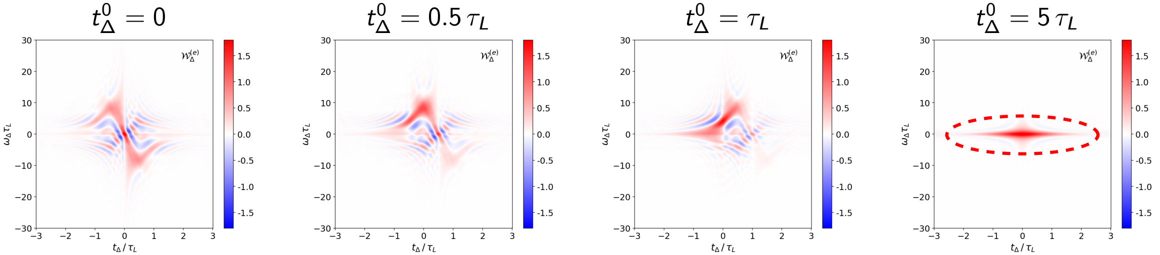
- Influence of the injection time offset t_Δ^0
  - $0 < t_\Delta^0 < \tau_L$: one energy exchange process is privileged (snowplough effect)
  - $t_\Delta^0 \gg \tau_L$: coherence collapses to the non-interacting case (quantum effects disappear)



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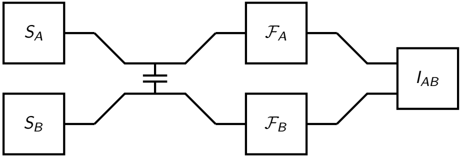

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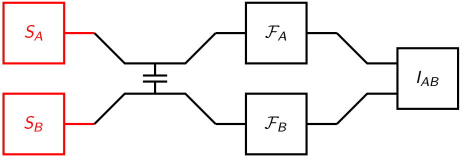

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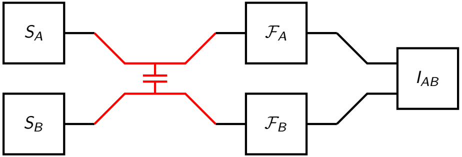

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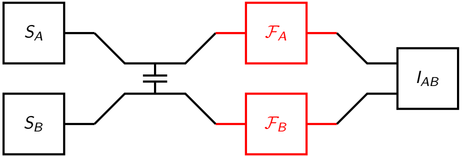

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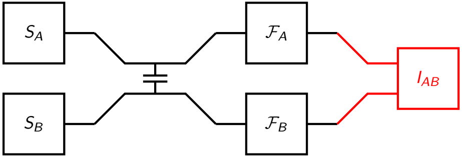
  - current correlations: reconstruct the coherence in the energy space



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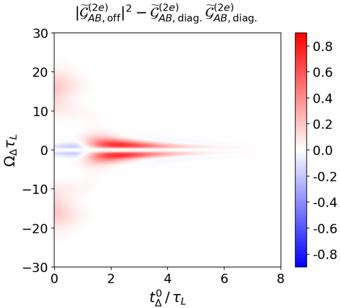

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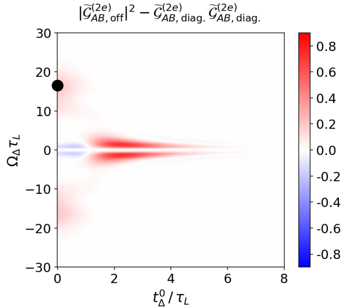

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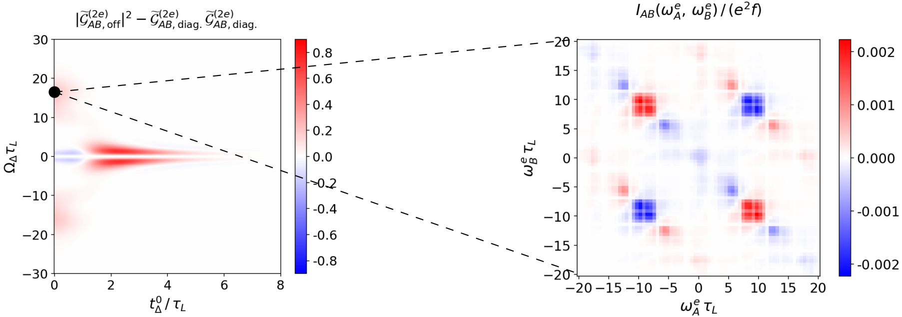
- Each point of $(t_\Delta^0, \Omega_\Delta) \implies$ signal in the whole (ω_A^e, ω_B^e) plane



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- Cauchy-Schwarz witness $\Longrightarrow$ detection of collision-induced entanglement
- Methods and formalism can be generalized to more realistic models
  - decoherence, exchanges with the environment, low-energy wavepackets
- How can one quantify the amount of created entanglement?
- Only one aspect of flying qubits dynamics: part of a more general approach!

Thank you for your attention!

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+ Choupette

+ Choupette