

Probing electronic entanglement *via* two-electron photo-assisted tomography

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ENS Lyon, Team 5 Coffee Seminar



Introduction

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- Two-electron tomography necessary to access entanglement *via* second-order coherence
- Two-electron tomography protocol + application to electronic entanglement detection

Outline

1. Electronic coherences and their representations

- 1.1 First-order electronic coherence function
- 1.2 Second-order electronic coherence function

2. Photo-assisted two-electron tomography

- 2.1 Two-electron tomography protocol
- 2.2 Reconstruction of the second-order coherence

3. Collision-induced electronic entanglement

- 3.1 Coherent collision of two Landau wavepackets
- 3.2 Signatures of electronic entanglement

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- **Time-domain** representation: access the **dynamics** of the system

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- Fourier-domain representation: nature of electronic excitations (content in energy)

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$$\mathcal{W}_{\hat{\rho}}^{(e)}(t | \omega) = v_F \int_{\mathbb{R}} d\tau \mathcal{G}_{\hat{\rho}}^{(e)}(t | \tau) e^{i\omega\tau} = v_F \int_{\mathbb{R}} \frac{d\Omega}{2\pi} \tilde{\mathcal{G}}_{\hat{\rho}}^{(e)}(\Omega | \omega) e^{-i\Omega t}$$

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$$\mathcal{G}_{AB}^{(2e)}(t_A^+, t_B^+ | t_A^-, t_B^-) = \langle \hat{\psi}_A^\dagger(t_A^-) \hat{\psi}_B^\dagger(t_B^-) \hat{\psi}_B(t_B^+) \hat{\psi}_A(t_A^+) \rangle_{\hat{\rho}_{AB}}$$

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- Overlap between two **two-electron** quantum paths $\implies$ **4D-function**
- Find convenient **3D representation** to build a physical intuition

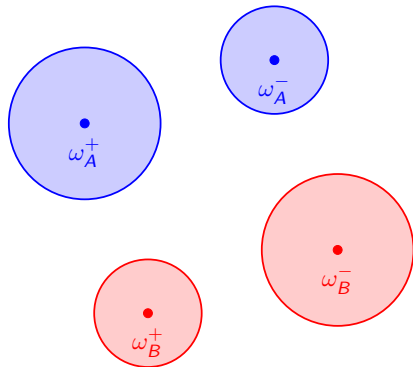
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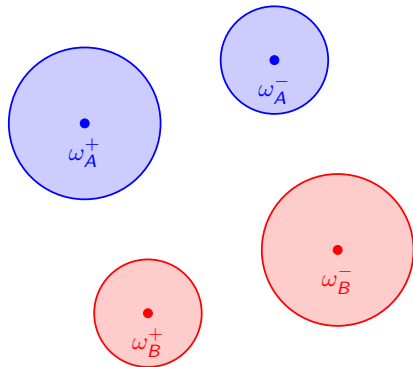
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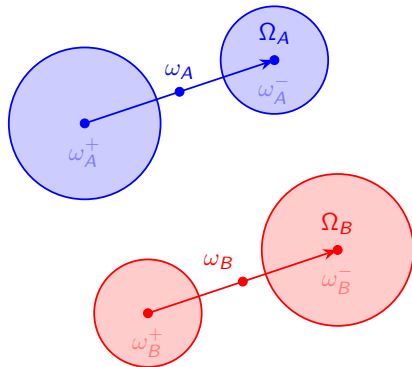
- Starting point: $\tilde{\mathcal{G}}_{AB}^{(2e)}(\omega_A^+, \omega_B^+ | \omega_A^-, \omega_B^-)$
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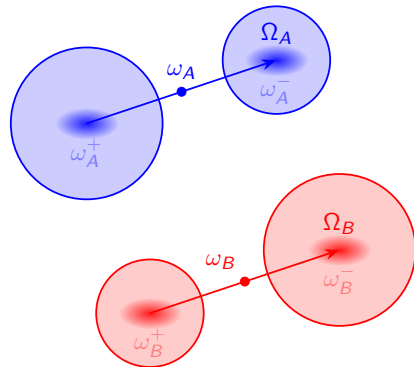
$$\begin{cases} \omega_A = \frac{1}{2}(\omega_A^+ + \omega_A^-) \\ \omega_B = \frac{1}{2}(\omega_B^+ + \omega_B^-) \end{cases} \quad \text{and} \quad \begin{cases} \Omega_A = \omega_A^+ - \omega_A^- \\ \Omega_B = \omega_B^+ - \omega_B^- \end{cases}$$



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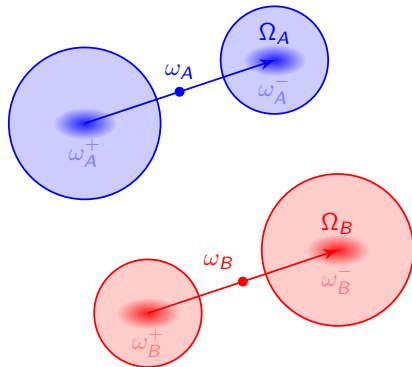


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- Center of mass and relative particle for Ω -variables



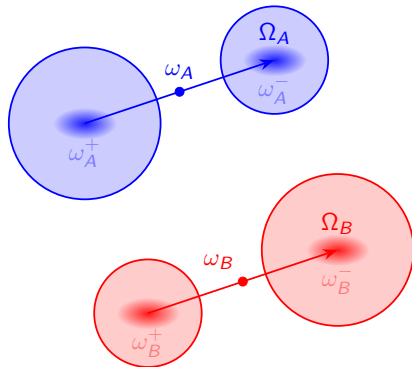
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$$\Omega_\Sigma = \Omega_A + \Omega_B \quad \text{and} \quad \Omega_\Delta = \frac{1}{2}(\Omega_A - \Omega_B)$$



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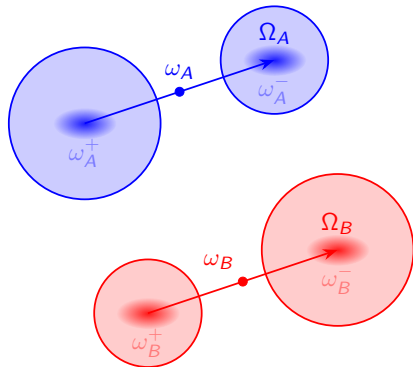
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- Center of mass and relative particle for Ω -variables

$$\Omega_\Sigma = \Omega_A + \Omega_B \quad \text{and} \quad \Omega_\Delta = \frac{1}{2}(\Omega_A - \Omega_B)$$

- New representation: $\tilde{\mathcal{G}}_{AB}^{(2e)}(\omega_A, \omega_B | \Omega_\Sigma, \Omega_\Delta)$



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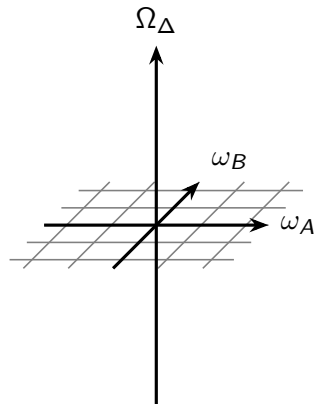
$$\tilde{\mathcal{G}}_{AB}^{(2e)}(\omega_A, \omega_B \mid \Omega_\Sigma, \Omega_\Delta) \mapsto \tilde{\mathcal{G}}_{AB,n}^{(2e)}(\omega_A, \omega_B \mid \Omega_\Delta)$$

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- 3D representation: $(\omega_A, \omega_B, \Omega_\Delta)$ space

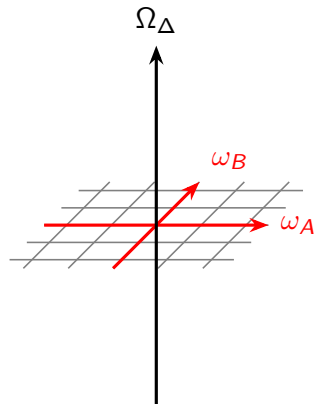


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- **3D representation:** $(\omega_A, \omega_B, \Omega_\Delta)$ space
 - ▶ (ω_A, ω_B) : **classical** coherence (**diagonal** plane)

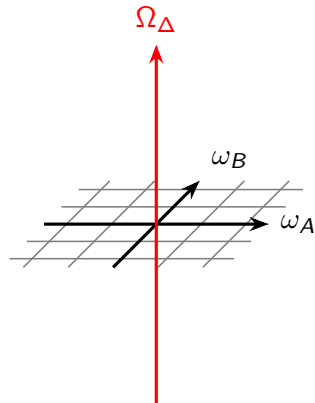


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- 3D representation: $(\omega_A, \omega_B, \Omega_\Delta)$ space
 - ▶ (ω_A, ω_B) : classical coherence (diagonal plane)
 - ▶ Ω_Δ : quantum coherence (off-diagonal line)



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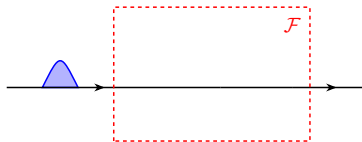
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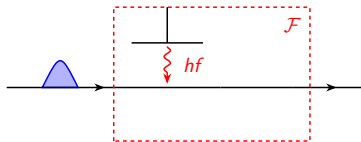
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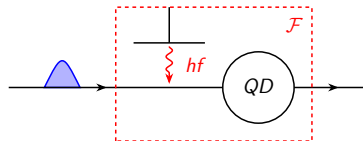
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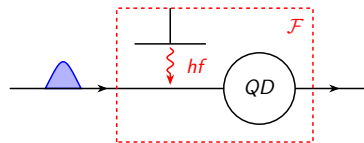
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- Access off-diagonal coherence: photo-assisted filtering
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 - ▶ quantum dot acting as an energy filter

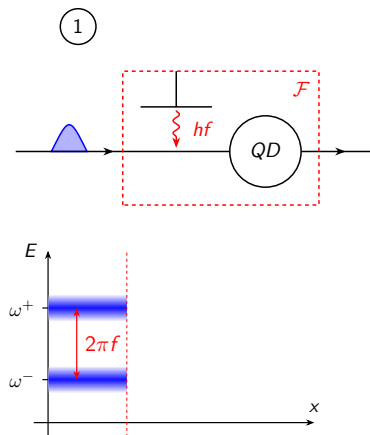
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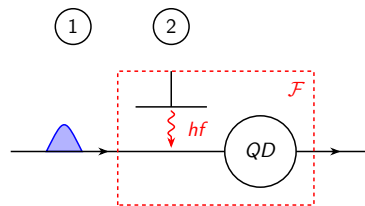


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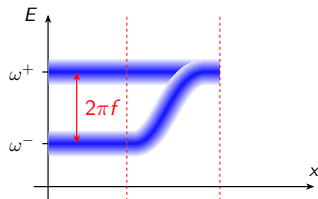


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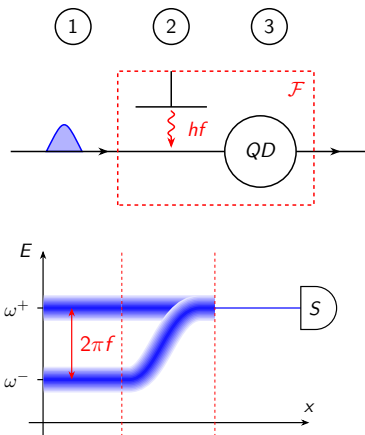
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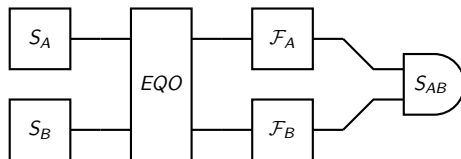
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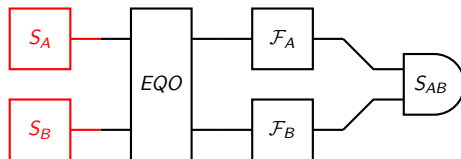
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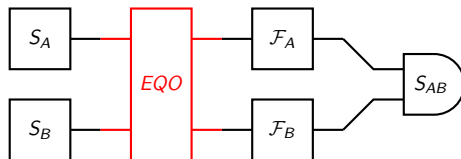
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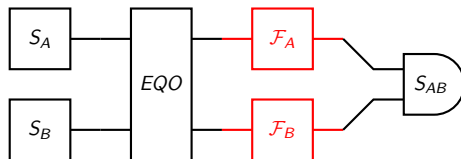
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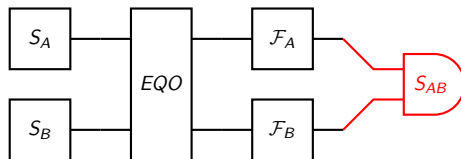
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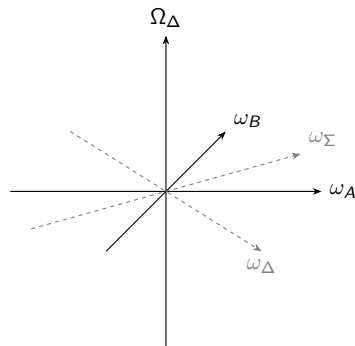
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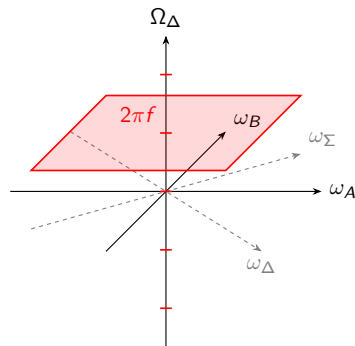
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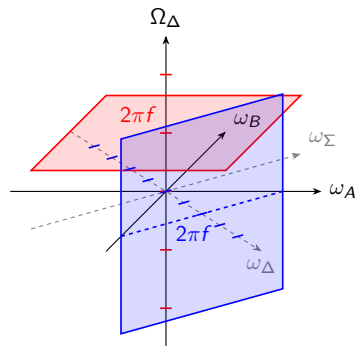
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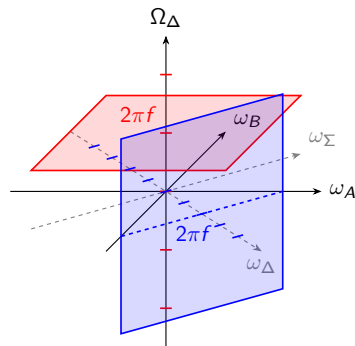
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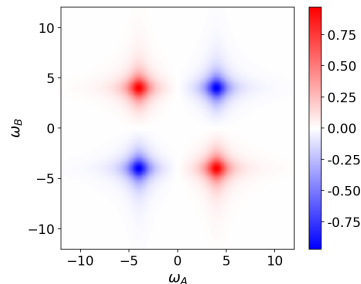
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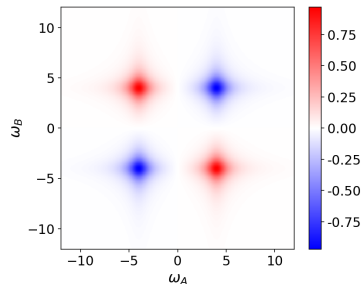
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- Wigner function of **connected two-electron coherence**: $\Delta_c \mathcal{W}_{AB}^{(2e)} = \mathcal{W}_{AB}^{(2e)} - \mathcal{W}_A^{(e)} \mathcal{W}_B^{(e)}$

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- $\Delta_W \mathcal{W}_{AB}^{(2e)}$ reconstructed with a Bayesian deconvolution algorithm (with physical prior)

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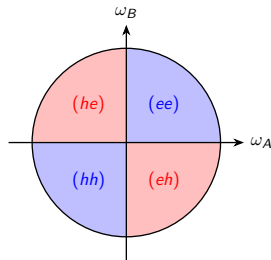
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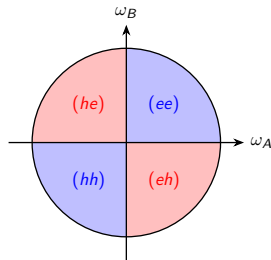
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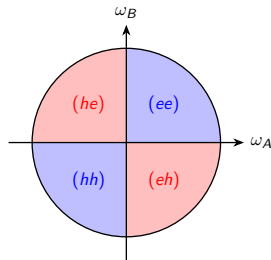
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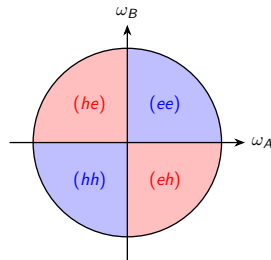
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- Allow to **constrain** $\Delta_W \tilde{\mathcal{G}}_{AB}^{(2e)}$ (in **certain basis**)



Outline

1. Electronic coherences and their representations

1.1 First-order electronic coherence function

1.2 Second-order electronic coherence function

2. Photo-assisted two-electron tomography

2.1 Two-electron tomography protocol

2.2 Reconstruction of the second-order coherence

3. Collision-induced electronic entanglement

3.1 Coherent collision of two Landau wavepackets

3.2 Signatures of electronic entanglement

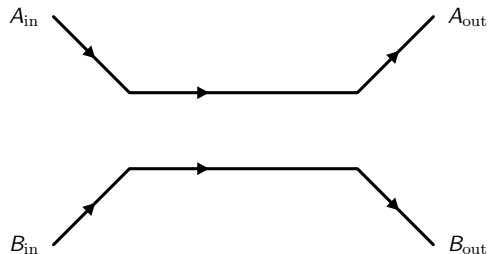
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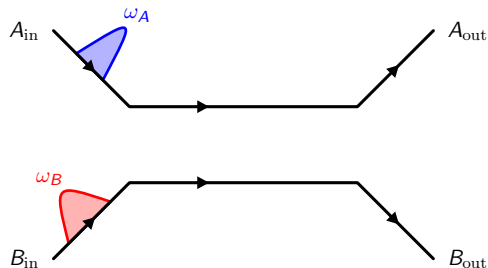
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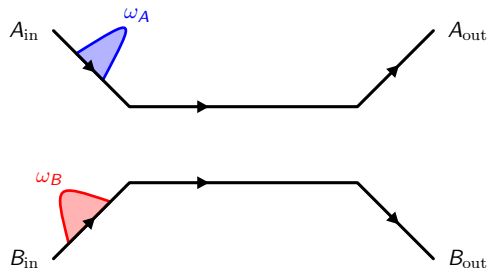
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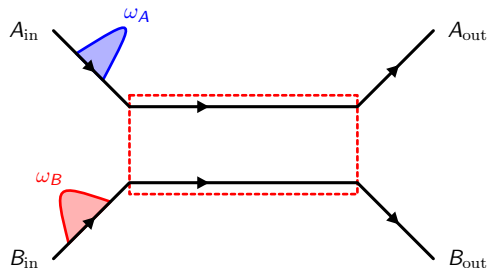
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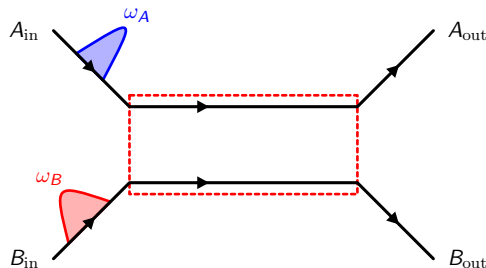
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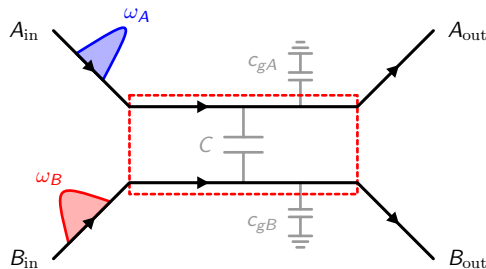
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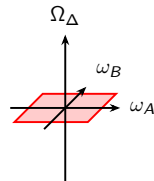
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- Diagonal coherence (energy probability distribution)

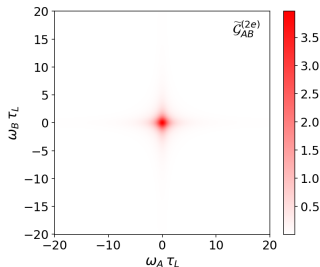
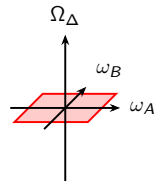
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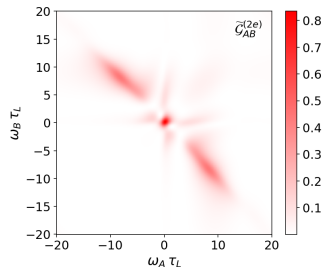


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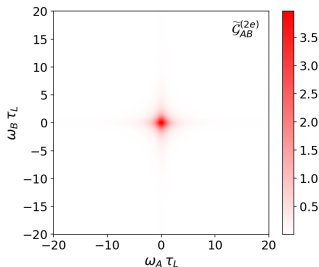
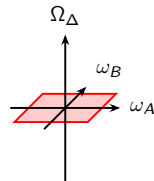


Collision →

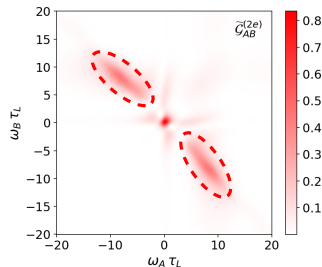


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 - ▶ lateral lobes: energy transfers $A \rightarrow B$ and $B \rightarrow A$

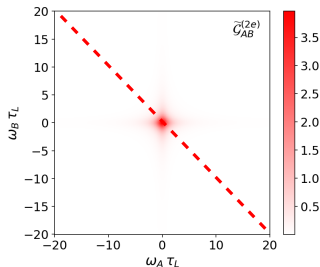
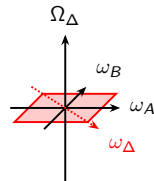


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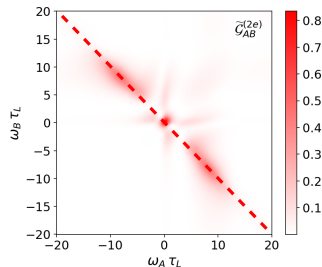


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- Diagonal coherence (energy probability distribution)
 - ▶ lateral lobes: energy transfers $A \rightarrow B$ and $B \rightarrow A$
 - ▶ spread along $\omega_\Delta = \frac{1}{2}(\omega_A - \omega_B)$: energy conservation

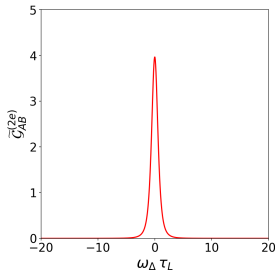
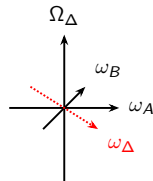


Collision →

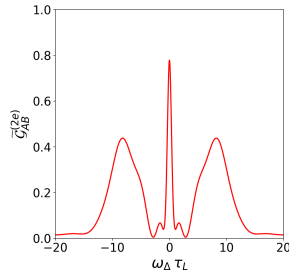


3.1. Coherent collision of two Landau wavepackets

- Diagonal coherence (energy probability distribution)
 - ▶ lateral lobes: energy transfers $A \rightarrow B$ and $B \rightarrow A$
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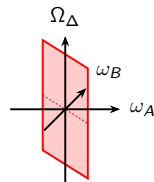
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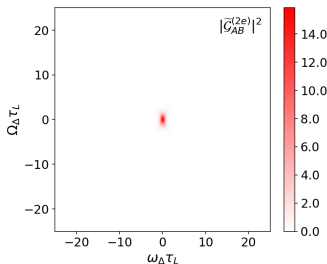
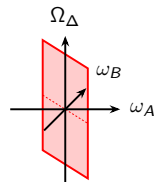
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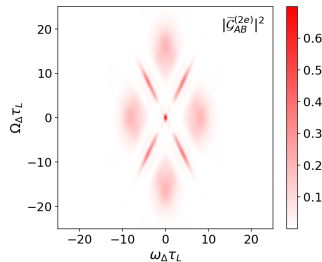


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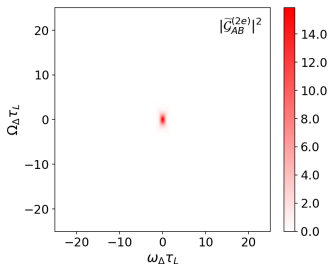
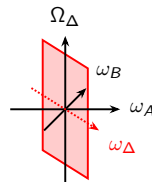


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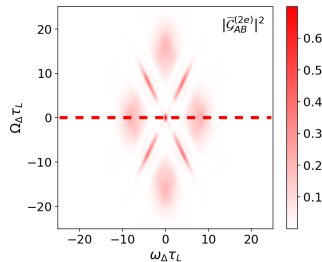


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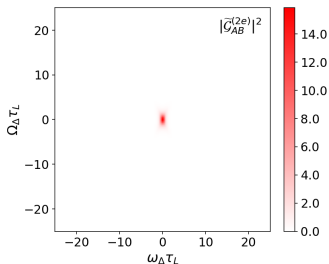
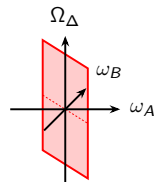


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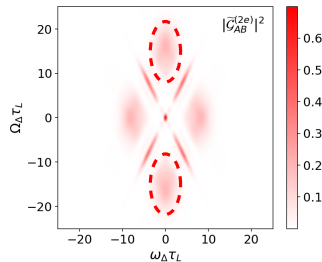


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- Off-diagonal coherence (quantum component)
 - ▶ vertical lobes: interferences between lateral lobes

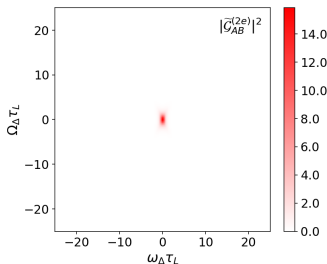
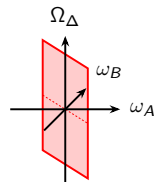


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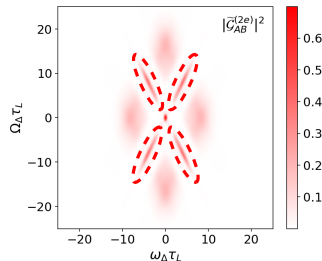


3.1. Coherent collision of two Landau wavepackets

- Off-diagonal coherence (quantum component)
 - ▶ vertical lobes: interferences between lateral lobes
 - ▶ fringes: interferences central structure / lateral lobes



Collision →



3.2. Signatures of electronic entanglement

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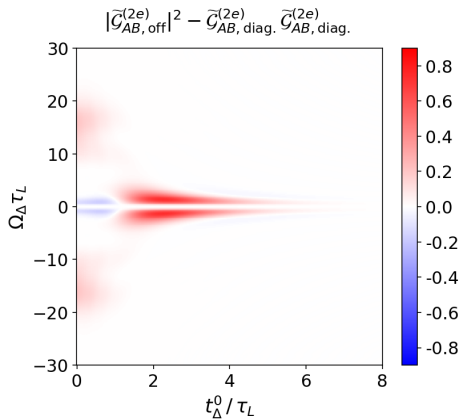
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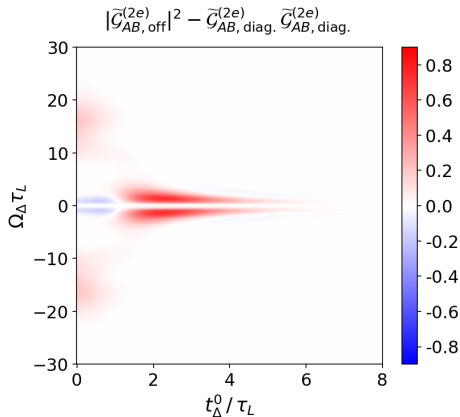
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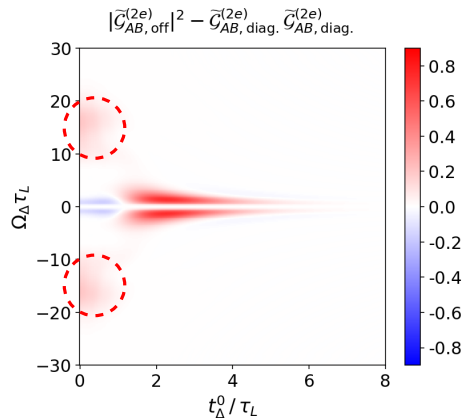
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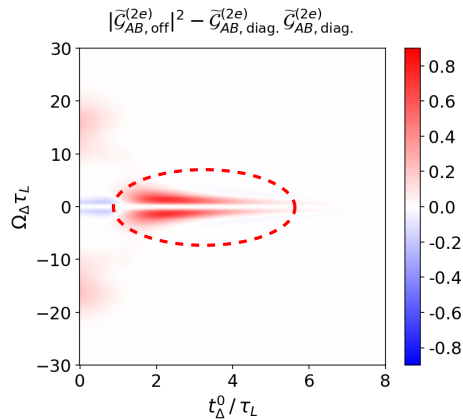
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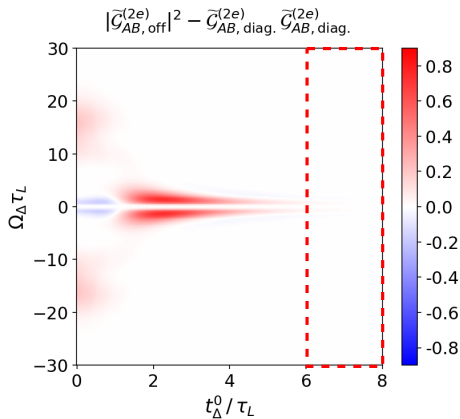
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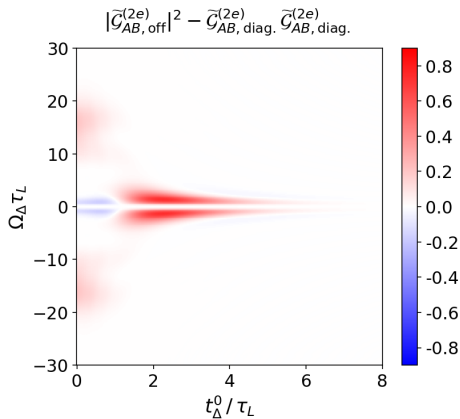
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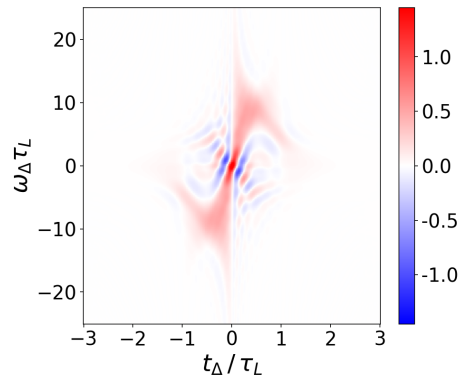
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- Wigner function for the relative particle: coherence in the 2D $(t_{\Delta}, \omega_{\Delta})$ plane

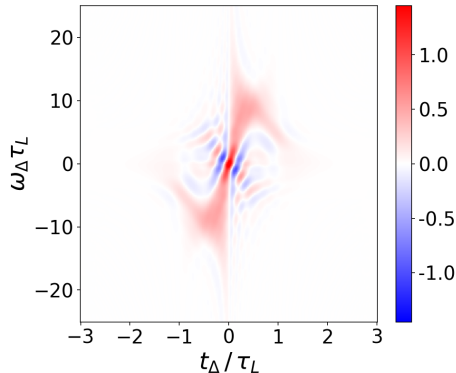
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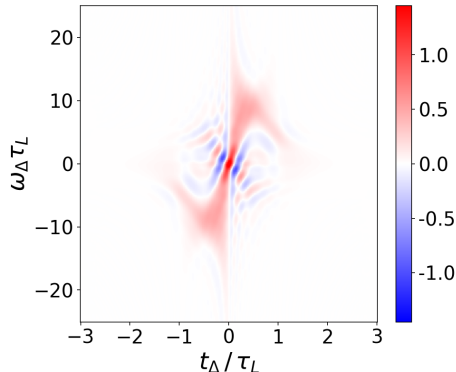
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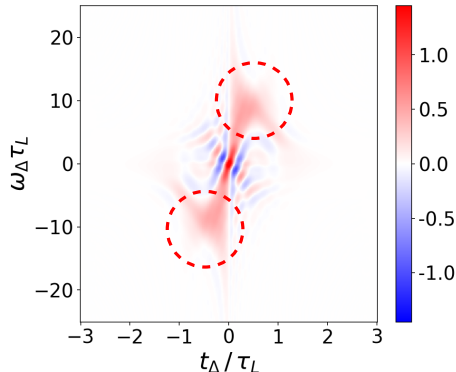
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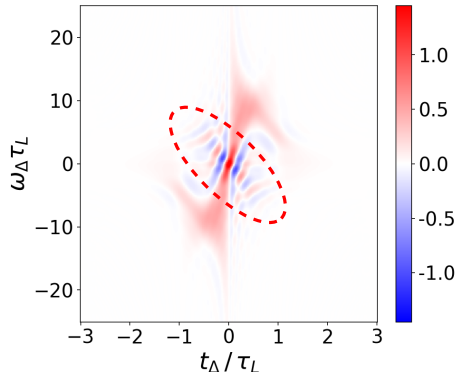
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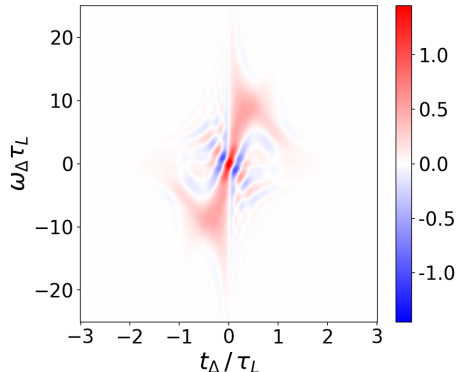
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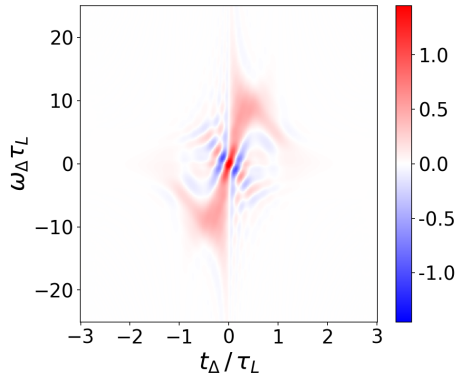
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- Wigner representation highlights entanglement!



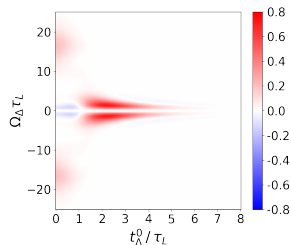
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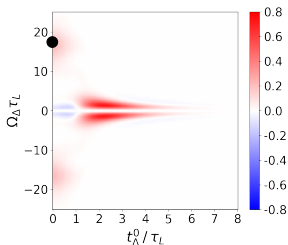
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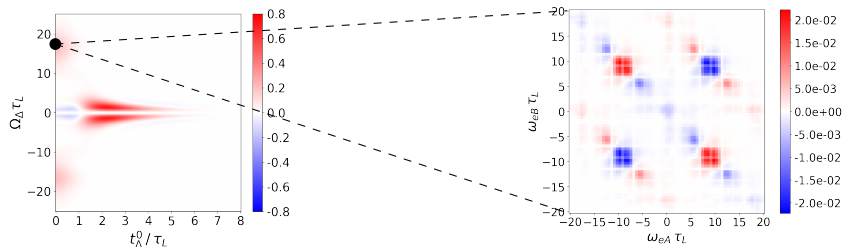
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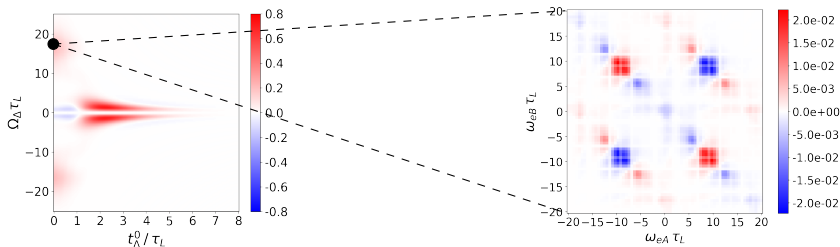
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- How can one **quantify** and **measure** the amount of **entanglement** created?

Thank you for your attention!



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A. Cauchy-Schwarz constraints

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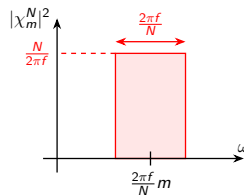
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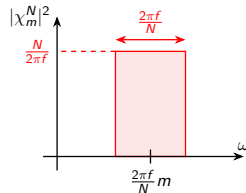
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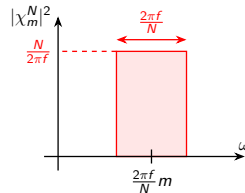
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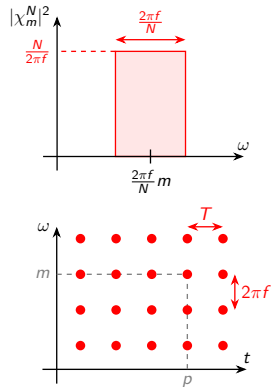
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► $\chi_m^N(\omega) = \sqrt{\frac{N}{2\pi f}} \mathbb{1}_{[0, \frac{2\pi f}{N}]}(\omega - \frac{2\pi f}{N} m) \exp(i \frac{\omega}{f})$

- not able to constrain $\Delta_W \tilde{\mathcal{G}}_{AB}^{(2e)}$: crushed by scaling

- Atoms of signal (Floquet-Wannier basis)

► $\chi_m^p = \frac{1}{\sqrt{f}} \int_0^{2\pi f} \frac{d\omega}{2\pi} \exp(-i \frac{\omega}{m} p) \psi_m(\omega), \psi$ Bloch wave



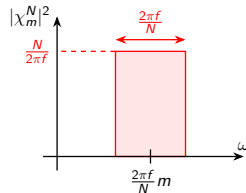
A. Cauchy-Schwarz constraints

- Regularization of $\tilde{\mathcal{G}}^{(2e)}$: $\tilde{\mathcal{G}}_{AB}^{(2e)}(\omega_A^+, \omega_B^+ | \omega_A^-, \omega_B^-) = \left\langle \chi_{\omega_A^-}, \chi_{\omega_B^-} \left| \hat{\mathcal{G}}^{(2e)} \right| \chi_{\omega_A^+}, \chi_{\omega_B^+} \right\rangle$

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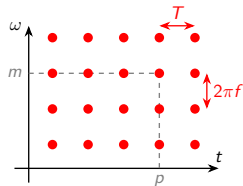
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- set of constraints on the regular part $\Delta_W \tilde{\mathcal{G}}_{AB}^{(2e)}$



B. Landau wavepackets

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- Landau wavepacket: emitted by single-electron source

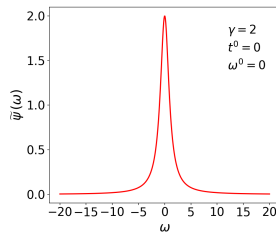
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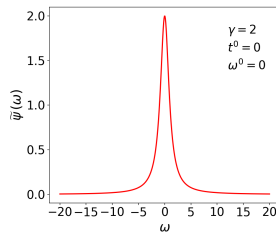
$$\tilde{\psi}(\omega) = \sqrt{\gamma} \frac{e^{-i\omega t^0}}{\omega - \omega^0 + i\frac{\gamma}{2}} \quad \text{with} \quad \begin{cases} \gamma \text{ width} \\ t^0 \text{ injection time} \\ \omega^0 \text{ center frequency} \end{cases}$$



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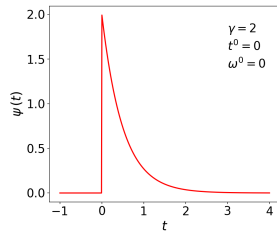
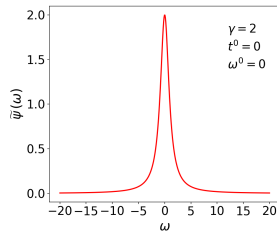
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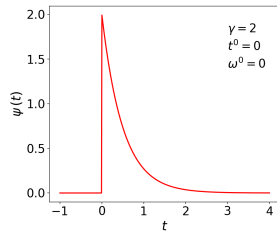
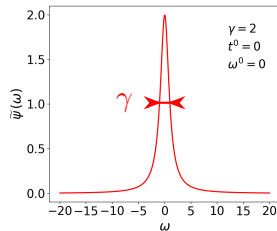
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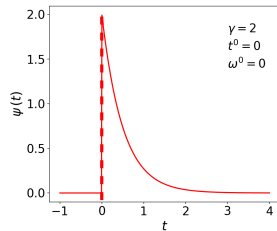
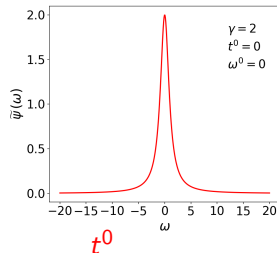
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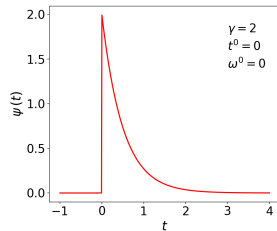
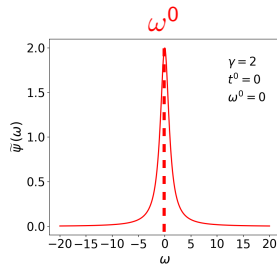
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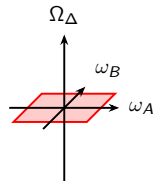
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C.1. Diagonal coherence

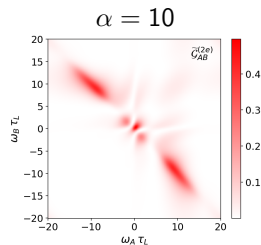
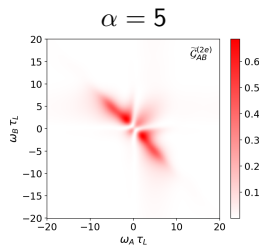
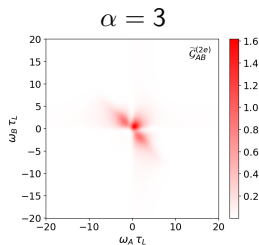
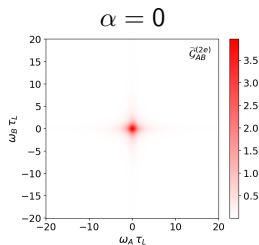
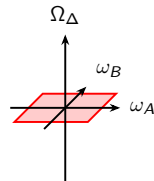
C.1. Diagonal coherence

- Evolution with coupling parameter α



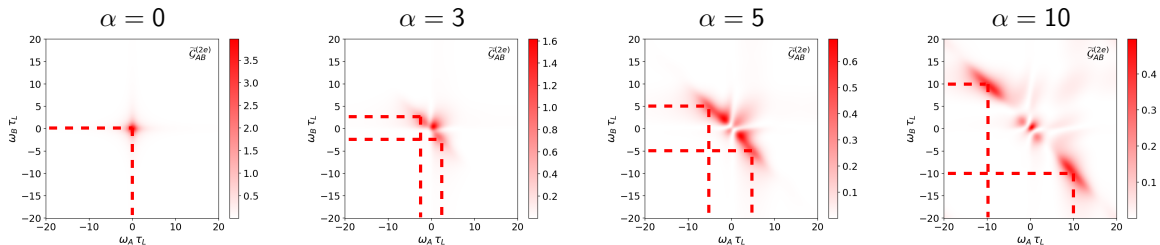
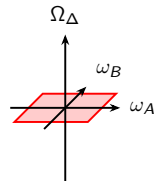
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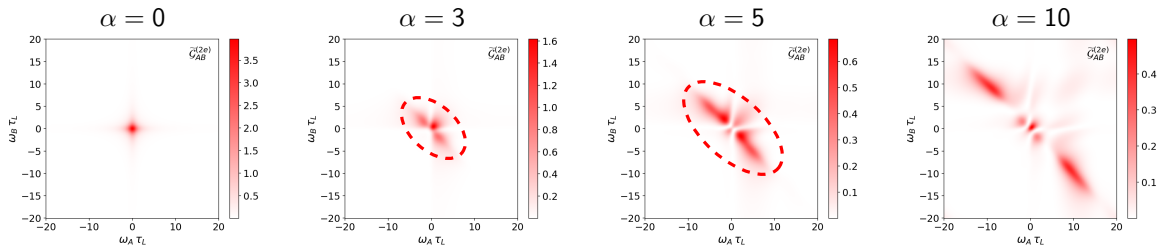
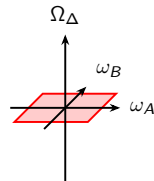
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 - lobes around $\omega_{\tau_L} = \pm\alpha$: exchanged energy increases with α



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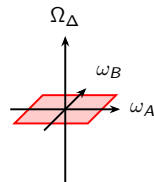
- Evolution with coupling parameter α
 - ▶ lobes around $\omega_{\tau_L} = \pm\alpha$: exchanged energy increases with α
 - ▶ α too weak $\Rightarrow$ lobes not clearly resolved



C.2. Diagonal coherence

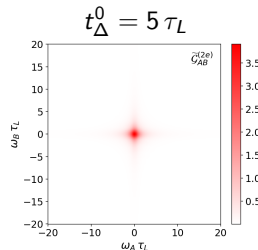
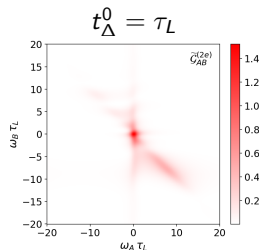
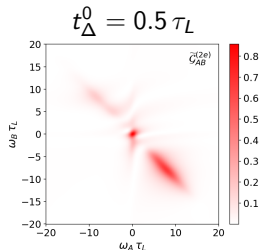
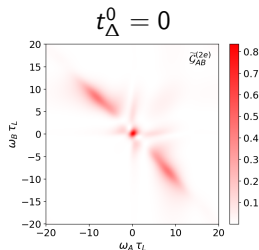
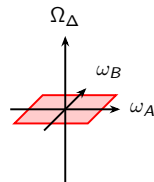
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- Influence of the injection time offset $t_{\Delta}^0 = t_A^0 - t_B^0$



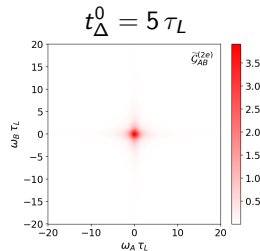
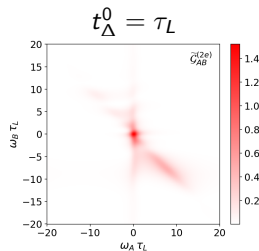
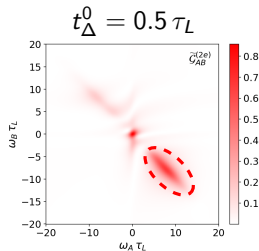
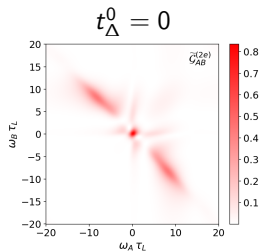
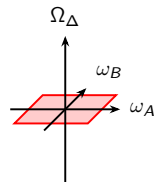
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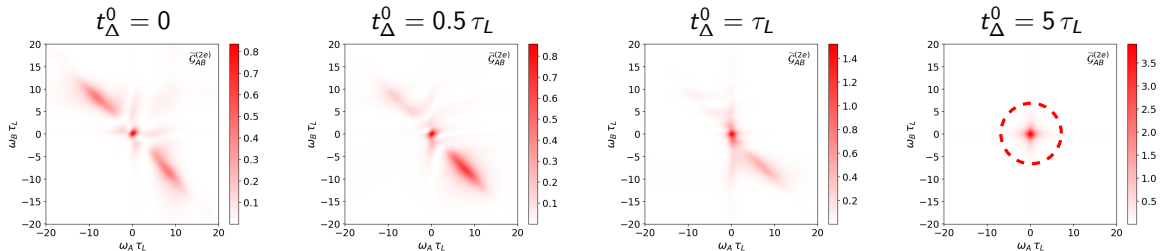
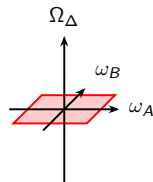
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C.2. Diagonal coherence

- Influence of the injection time offset $t_{\Delta}^0 = t_A^0 - t_B^0$
 - ▶ $0 < t_{\Delta}^0 < \tau_L \implies$ one transfer preferred over the other
 - ▶ $t_{\Delta}^0 > \tau_L$: electrons don't see each other ($\tilde{\mathcal{G}}_{\hat{\rho}_{AB,out}}^{(2e)} = \tilde{\mathcal{G}}_{\hat{\rho}_{AB,in}}^{(2e)}$)



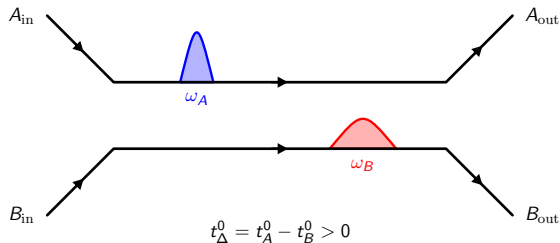
D. Snowplough effect

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- The late electron **takes energy** from the first electron

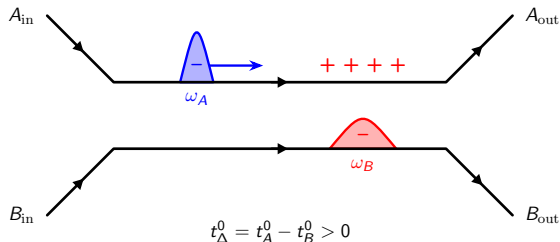
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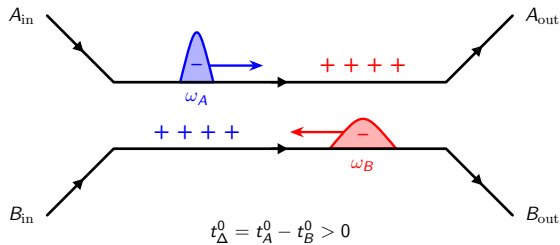
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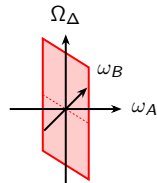
- The late electron **takes energy** from the first electron
 - ▶ first electron **polarizes** other channel $\implies$ **accelerates second electron**
 - ▶ reverse process: second electron **slows first electron down**



E.1. Off-diagonal coherence

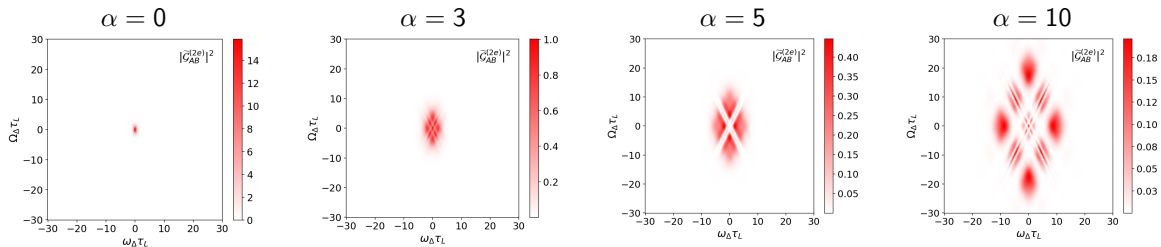
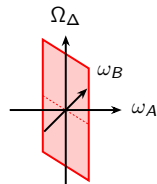
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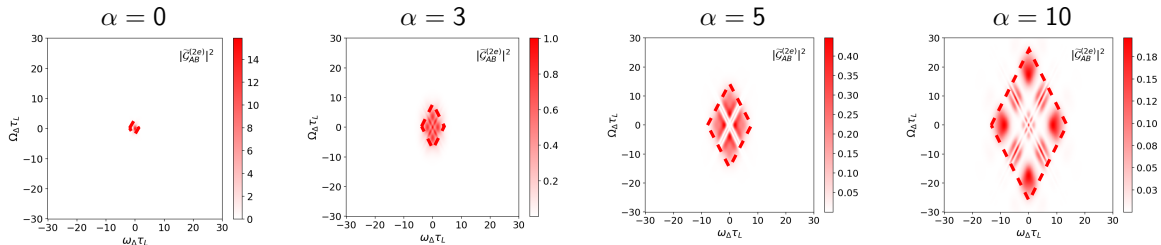
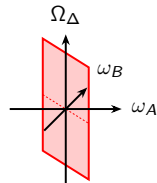
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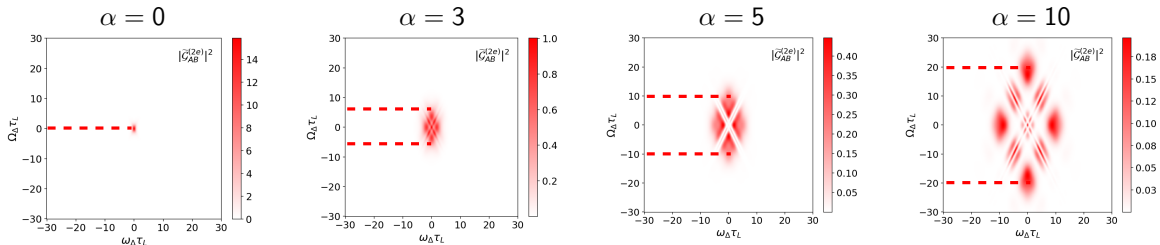
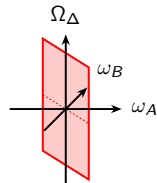
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 - ▶ diamond shape growing with α : Cauchy-Schwarz inequality



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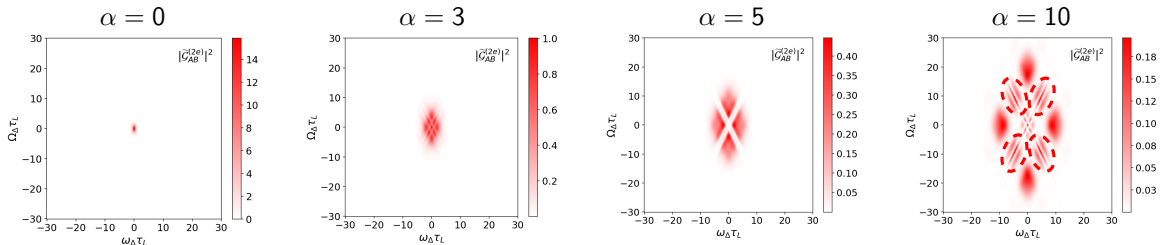
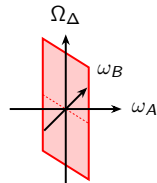
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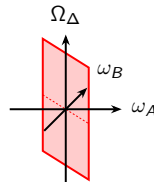
- ▶ diamond shape growing with α : Cauchy-Schwarz inequality
- ▶ vertical lobes at $\Omega_\Delta \tau_L = \pm 2\alpha$: quantum component $\propto$ coupling
- ▶ more interference fringes for high α : more exchange processes



E.2. Off-diagonal coherence

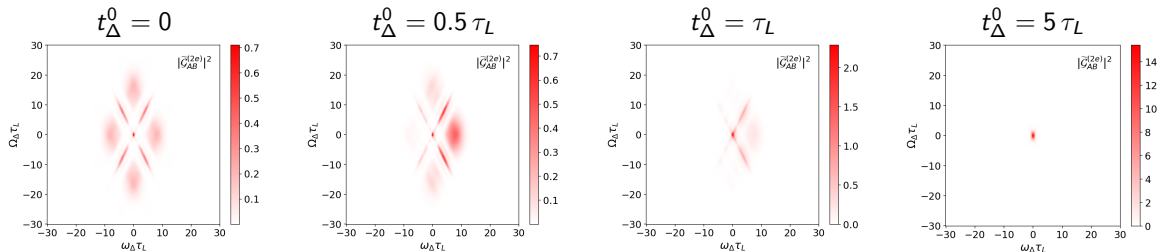
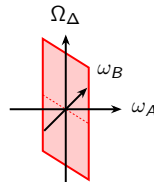
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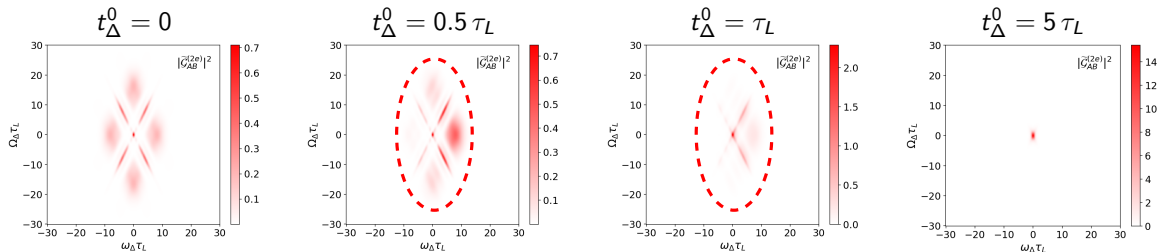
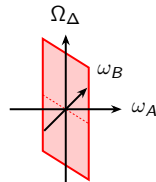
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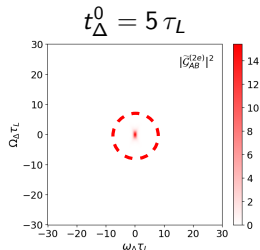
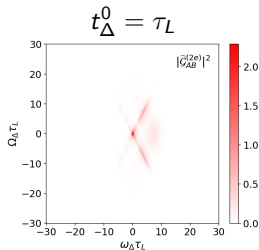
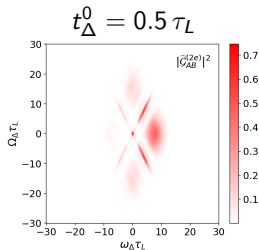
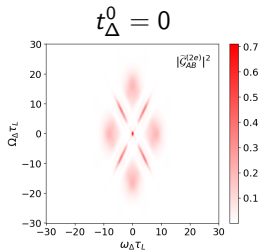
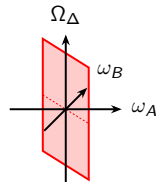
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 - ▶ snowplough effect $\implies$ fading of corresponding lobes and fringes



E.2. Off-diagonal coherence

- Influence of the injection time offset t_{Δ}^0
 - ▶ snowplough effect $\implies$ fading of corresponding lobes and fringes
 - ▶ $t_{\Delta}^0 \gg \tau_L$: coherence collapses to its input value ($\iff \alpha = 0$)



F.1. Entanglement witness

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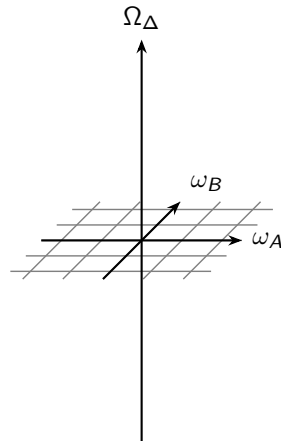
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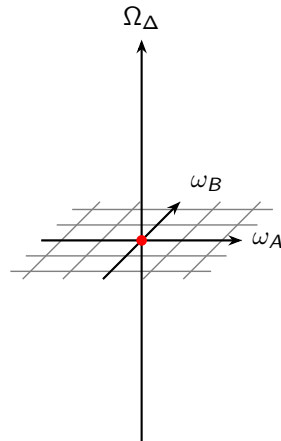
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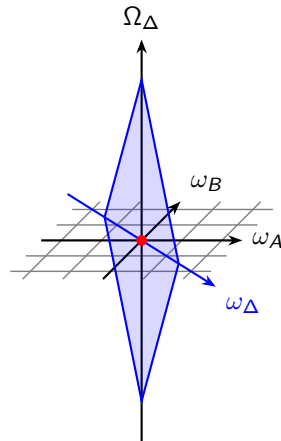
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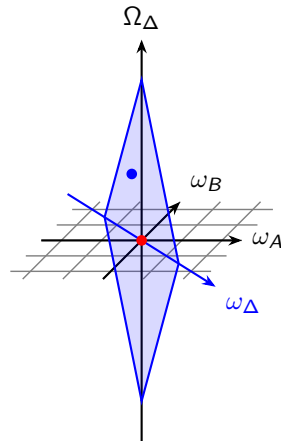
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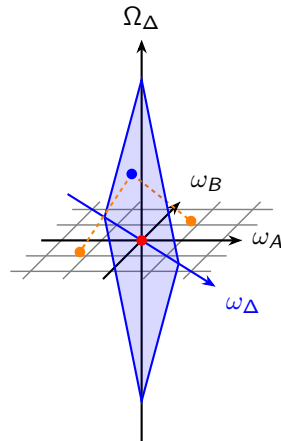
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 - ▶ witness l.h.s. (**output state**): anywhere in the **coherence diamond**



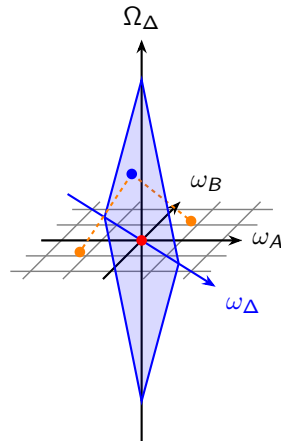
F.1. Entanglement witness

- Coherence is zero **outside of Cauchy-Schwarz domain** (inequality)
- Geometric **interpretation** of the witness
 - ▶ **input state**: centered at (ω_A^0, ω_B^0) in the **diagonal plane**
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- R.h.s. points **outside of the diamond** $\implies$ witness **violated**



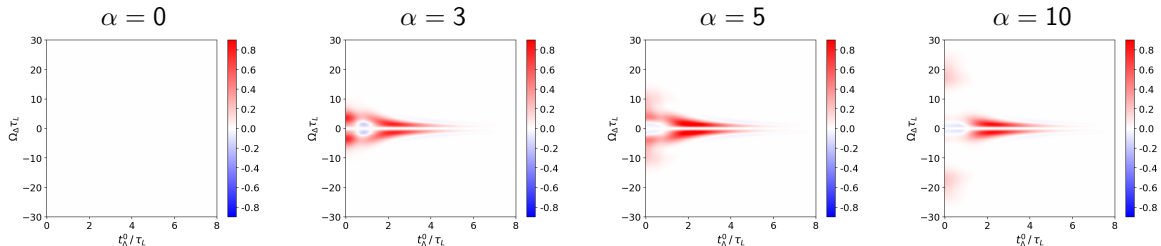
F.2. Entanglement witness

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- Evolution with coupling parameter α

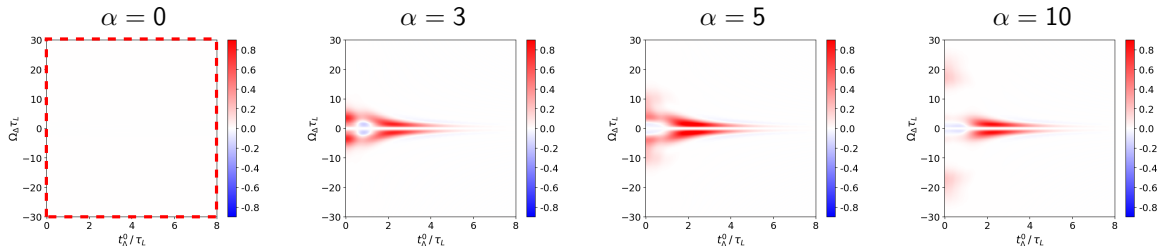
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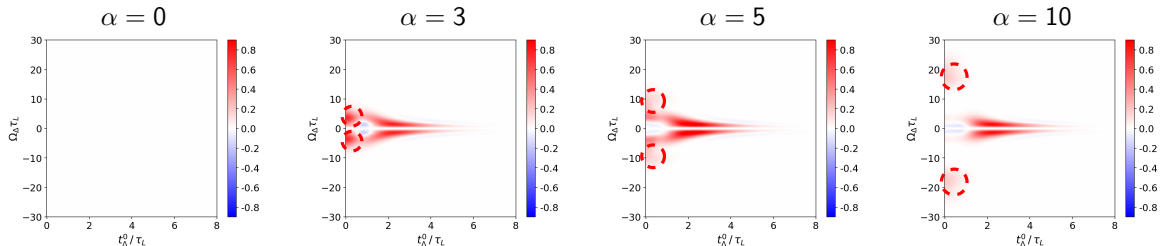
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 - ▶ $\alpha = 0$: no interaction $\implies$ both members of the witness are equal



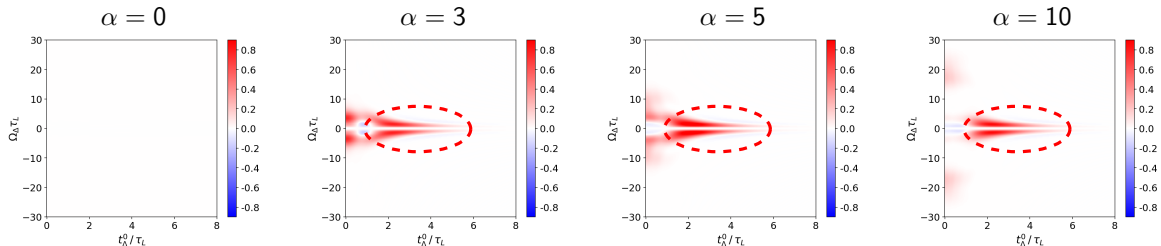
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 - ▶ nearly-diagonal violation region very resilient: present for all non-zero coupling strengths



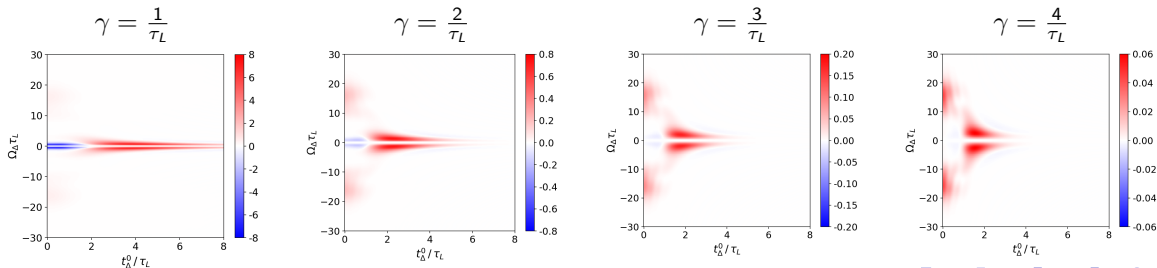
F.3. Entanglement witness

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- Evolution with wavepackets' spectral width γ (wavepacket's duration: $\tau_0 = \frac{1}{\gamma}$)

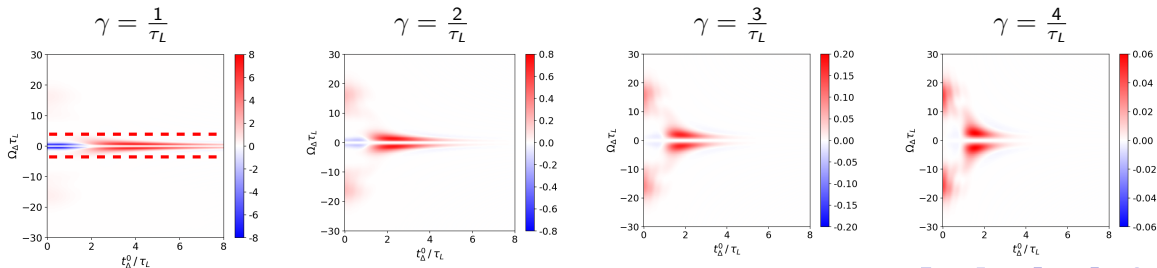
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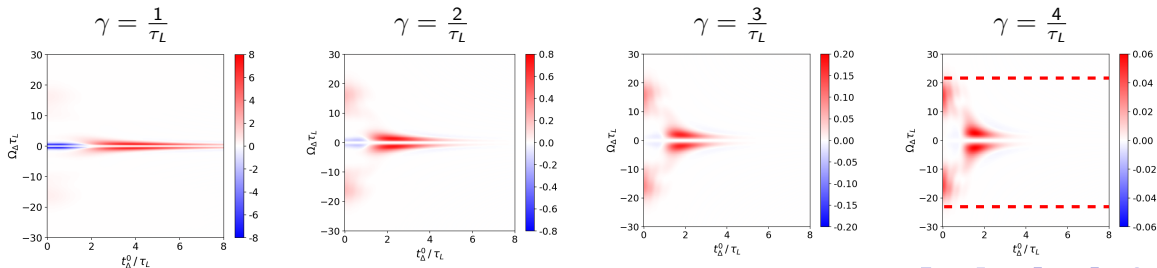
F.3. Entanglement witness

- Evolution with wavepackets' **spectral width** γ (wavepacket's duration: $\tau_0 = \frac{1}{\gamma}$)
 - ▶ **long** wavepackets (small γ) $\implies$ coherence concentrated **around diagonal plane**



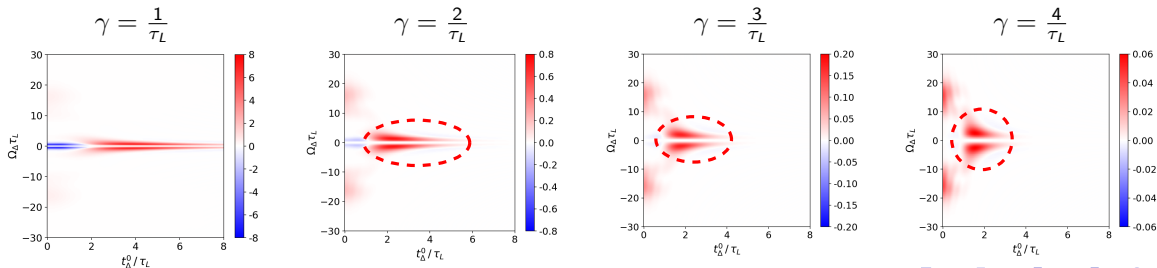
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 - ▶ **long** wavepackets (small γ) $\implies$ coherence concentrated **around diagonal plane**
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 - ▶ increasing γ limits **temporal overlap** between wavepackets $\implies$ signal fades quicker



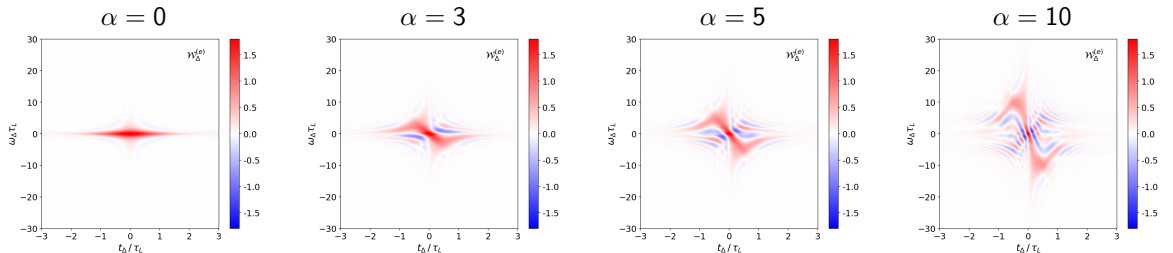
G.1. Wigner transform

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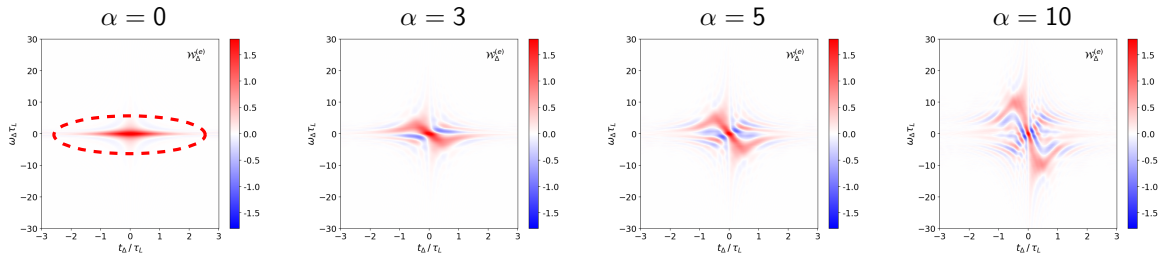
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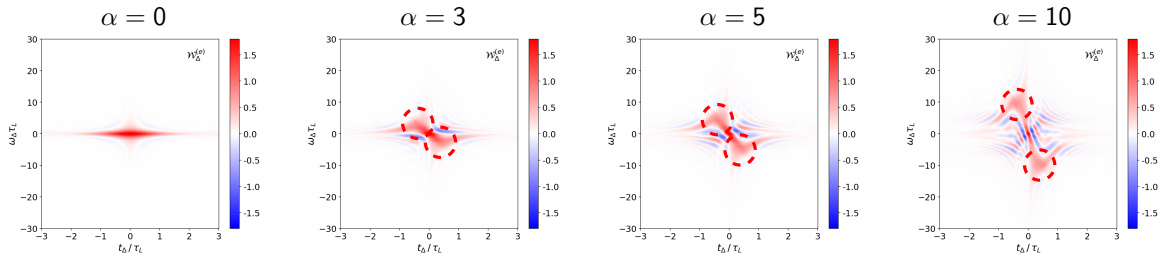
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► $\alpha = 0$: no interactions $\implies$ diagonal coherence and no negativities



G.1. Wigner transform

- Evolution with coupling parameter α
 - ▶ $\alpha = 0$: no interactions $\implies$ diagonal coherence and no negativities
 - ▶ positive lobes around $\omega_{\Delta}\tau_L = \pm\alpha$: transferred energy increases with α



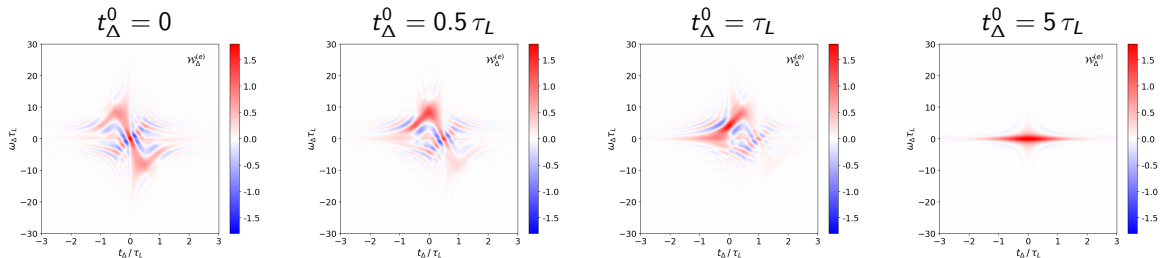
G.2. Wigner transform

G.2. Wigner transform

- Influence of the injection time offset t_{Δ}^0

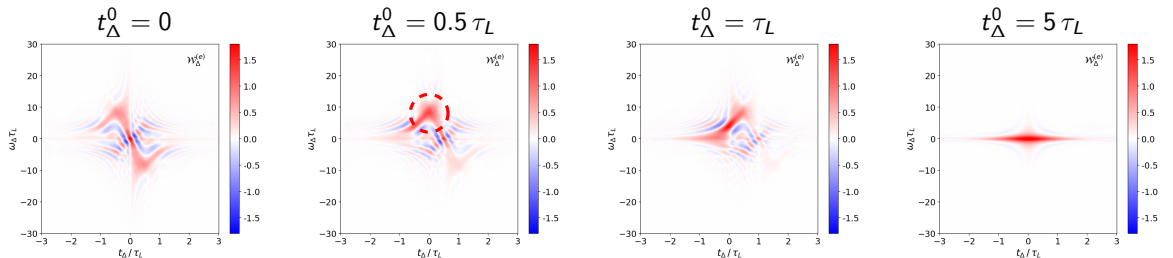
G.2. Wigner transform

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 - ▶ $0 < t_{\Delta}^0 < \tau_L$: one energy exchange process is privileged (snowplough effect)



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- Influence of the injection time offset t_{Δ}^0
 - ▶ $0 < t_{\Delta}^0 < \tau_L$: one energy exchange process is privileged (snowplough effect)
 - ▶ $t_{\Delta}^0 \gg \tau_L$: coherence collapses to the non-interacting case (quantum effects disappear)

