Entanglement in Electron Quantum Optics

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Motivations

- Using quantum optics tools to describe entanglement in electron quantum optics circuits [1].
- Correlations associated to Fermi statistics and Coulomb interactions.
- *Collisional experiments* to extract information about two-particle excitations.

Quantum information with fermions

Entanglement for indistinguishable excitations [2]:

- Based on mode separation: $\mathcal{H}^{(1p)} = \mathcal{H}_A \overset{\perp}{\oplus} \mathcal{H}_B$
- Separable states in Fock space $\mathcal{F}$:

$$|\psi\rangle_{sep} = P_A(a_1^\dagger, \ldots, a_p^\dagger) P_B(b_1^\dagger, \ldots, b_q^\dagger)\,|0\rangle$$

- Entangled states are the non-separable ones.

Physical state space and mode composition:

- **Bosons**: $\mathcal{H}_{\text{phys}} = \mathcal{F}_B$. Alice and Bob modes composed by $\otimes$, the tensor product, i.e. $\mathcal{F}_B = \mathcal{F}_B^{(\text{Alice})} \otimes \mathcal{F}_B^{(\text{Bob})}$.
- **Fermions**: obey Fermi statistics so Alice and Bob modes composed with $\wedge$, **the exterior product**, i.e. $\mathcal{F}_F = \mathcal{F}_F^{(\text{Alice})} \wedge \mathcal{F}_F^{(\text{Bob})}$.
  And they obey the **parity superselection rule**: $\mathcal{H}_{\text{phys}} \neq \mathcal{F}_F$ but [3]:

$$\mathcal{H}_{\text{phys}} = \mathcal{F}_F^{\text{even}} \cup \mathcal{F}_F^{\text{odd}}$$

Resources inequalities:

$$[f] \geq [c] \quad \text{but} \quad \neg([f] \geq [q])$$
$$2[f] \geq [q] \quad \text{and} \quad [q] \geq 2[f]$$

Quantum teleportation of a fermionic mode:

$$[ff] + 2[c \to c] + [f]_B \geq [f_E f_A \to f_E f_B]$$
$$[qq] + 2[c \to c] \geq [q \to q] \quad \text{(Qubit)}$$

Entanglement witness

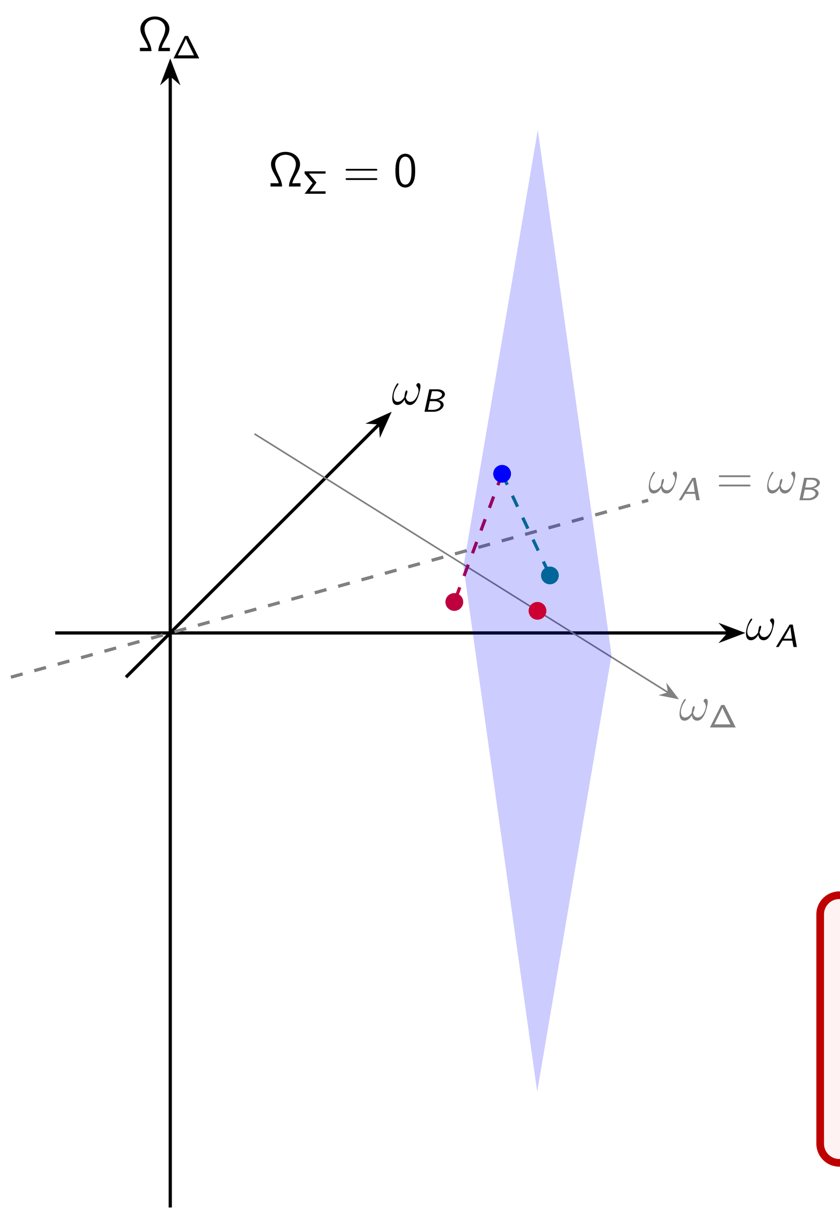
Cauchy-Schwarz entanglement witness on **two-electron coherence** [4]:

$$\left|\widetilde{\mathcal{G}}_{AB}^{(2e)}(\omega_1, \omega_2 \,|\, \omega_1', \omega_2')\right|^2 \leq \widetilde{\mathcal{G}}_{AB}^{(2e)}(\omega_1, \omega_2 \,|\, \omega_1, \omega_2)\, \widetilde{\mathcal{G}}_{AB}^{(2e)}(\omega_1', \omega_2' \,|\, \omega_1', \omega_2')$$

- Compares one **off-diagonal** coherence with two **diagonal** coherences.
- **Witness**: (violated $\Longrightarrow$ entangled) **but** (respected $\not\Longrightarrow$ separated).
- Clicks for **non-Positive Partial Transposed** (non-PPT) states:
  ρ is separable $\Longrightarrow$ **all** the eigenvalues of ${}^{t_A}\rho$ and ${}^{t_B}\rho$ are **positive**.
- Sensitive to **energy-bin entanglement** (see Ref. [5] in microwave quantum optics).

Collision induced entanglement

Coherent collision of two perfectly **energy-localized electrons**:



- **4D-space** split into two 2D-spaces:
  - (ω_A, ω_B): **diagonal** coherence $\Longleftrightarrow$ **classical** variables
  - $(\Omega_\Sigma, \Omega_\Delta)$: **off-diagonal** coherence $\Longleftrightarrow$ **quantum** variables.
- **Diamond**: area of non-zero coherence (**quantum scattering** contributions).
- To respect the witness, both C.S. points must be **in the diamond**

Quantum coherent energy transfer during collision $\Longrightarrow$ witness clicks

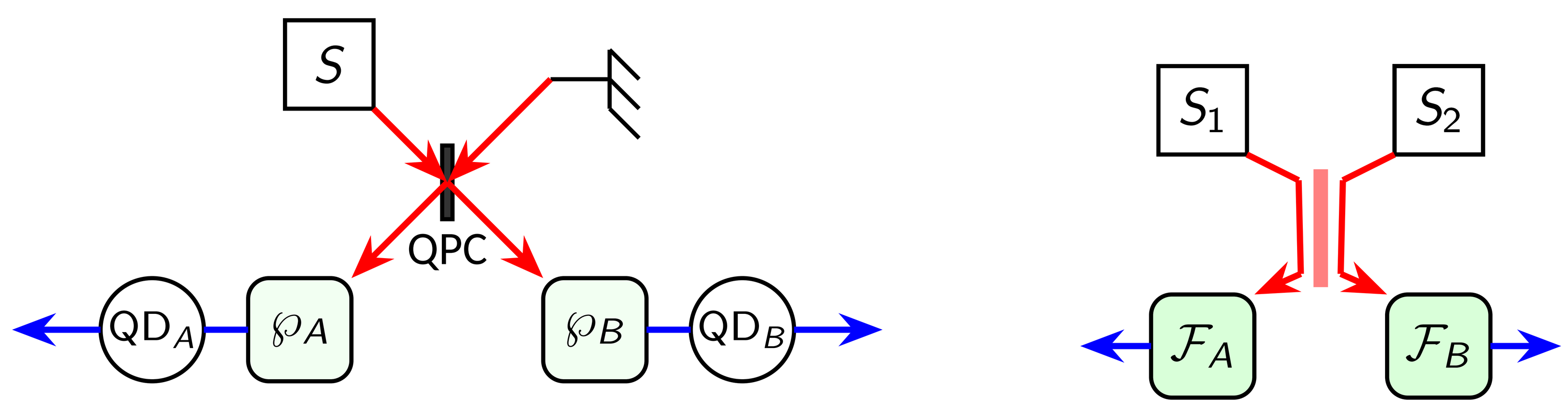
Next steps: departure from ideality

- discuss **energy-spread** electrons (Landau [6] or Leviton [7] excitations)
- Effect of **dissipative** collisions (**environmental effects**)

Two-electron coherence measurement

A **generalized Franson interferometer** [8] gives access to the two-electron coherence $\Delta\mathcal{G}^{(2e)}$ through current–current correlations:

$$\langle i_A i_B \rangle = RT\,(\mathcal{F}_A \otimes \mathcal{F}_B)\,\Delta\mathcal{G}_S^{(2e)}, \qquad \mathcal{F}_j = \text{QD}_j \circ \wp_j$$



- Photoassisted filters $\wp_{A,B}$ mix energies $\to$ access to **off-diagonal** components of $\Delta\mathcal{G}^{(2e)}$.
- Quantum dots $\text{QD}_{A,B}$ select energy $\to$ access to **diagonal** components.
- A combined filter/detector $\mathcal{F} = \text{QD}(\omega_e) \circ \wp\,[(p_n)_n]$ defines

$$\Pi[\mathcal{F}] = \sum_{n_+, n_-} p_{n_+} p_{n_-}^* \,|\omega_e - 2\pi n_+ f\rangle\langle\omega_e - 2\pi n_- f|,$$

  so that $P_{\text{click}} = \text{Tr}\left[\rho^{(1)}\Pi[\mathcal{F}]\right]$ contains both diagonal (via ω_e) and off-diagonal (via the $p_{n\neq 0}$) frequency components.
- Photoassisted filtering enables full **two-electron tomography**.

References

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