Collision-induced entanglement

Study of a two-particle coherent scattering model

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Summary

I. **Electronic coherence, entanglement witness and scattering model**

II. **Collision of two energy-localized wave-packets**

III. **Collision of two Landau wave-packets** *(work in progress)*

I. Electronic coherence, entanglement witness and scattering model

Second-order electronic coherence function

Electronic coherence, entanglement witness and scattering model

- $\mathcal{G}^{(2e)}$-function: $\mathcal{G}^{(2e)}_{\hat{\rho}}(t_A, t_B; t'_A, t'_B) = \langle \hat{\psi}^{\dagger}_A[t'_A]\, \hat{\psi}^{\dagger}_B[t'_B]\, \hat{\psi}_B[t_B]\, \hat{\psi}_A[t_A] \rangle_{\hat{\rho}}$
- In the **frequency space:** $\hat{\psi}^{\dagger}[t] := \int_{\mathbb{R}} \frac{d\omega}{\sqrt{2\pi v_F}}\, \hat{c}^{\dagger}[\omega]\, e^{-i\omega t}$

$$\widetilde{\mathcal{G}}^{(2e)}_{\hat{\rho}}(\omega_A, \omega_B; \omega'_A, \omega'_B) := v_F^2 \int_{\mathbb{R}^4} dt_A\, dt_B\, dt'_A\, dt'_B\, \mathcal{G}^{(2e)}_{\hat{\rho}}(t_1, t_2; t'_1, t'_2)\, e^{i(\omega_A t_A + \omega_B t_B - \omega'_A t'_A - \omega'_B t'_B)}$$

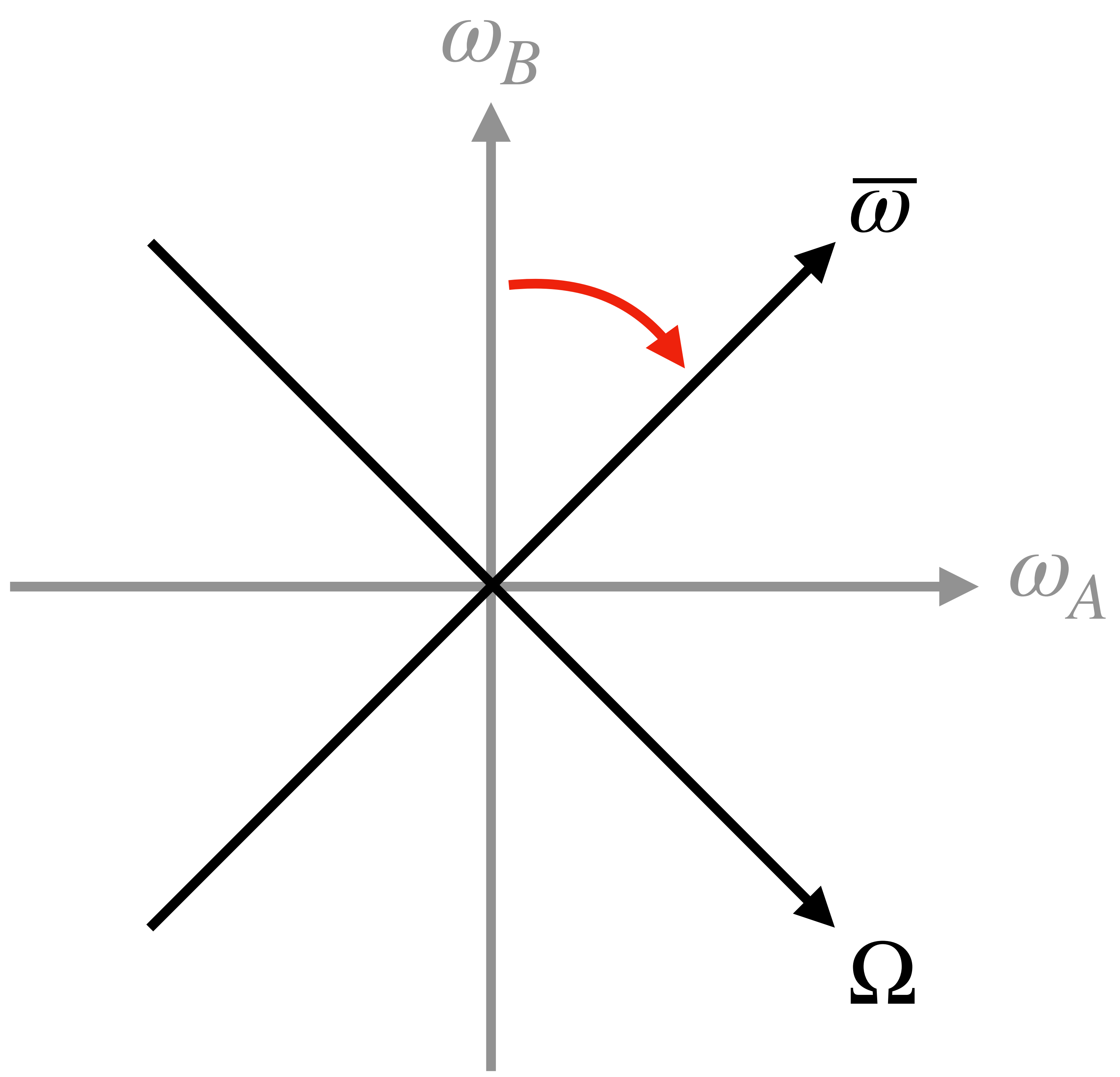
$$v_F^2\, \widetilde{\mathcal{G}}^{(2e)}_{\hat{\rho}}(\omega_A, \omega_B; \omega'_A, \omega'_B) := (2\pi)^2 \langle \hat{c}^{\dagger}_A[\omega'_A]\, \hat{c}^{\dagger}_B[\omega'_B]\, \hat{c}_B[\omega_B]\, \hat{c}_A[\omega_A] \rangle_{\hat{\rho}}$$

Diagonal and off-diagonal coordinates

Electronic coherence, entanglement witness and scattering model

- **First-order** coherence $\widetilde{\mathscr{G}}^{(e)}_{\hat{\rho}}(\omega_A;\omega_B)$
  - Change of variables: $(\overline{\omega},\Omega) = \left(\frac{\omega_A+\omega_B}{2}, \omega_A-\omega_B\right)$
  - **Average** energy $\overline{\omega}$: **diagonal** axis
  - Energy **difference** Ω**: off-diagonal** axis

$$\widetilde{\mathscr{G}}^{(e)}_{\hat{\rho}}(\omega_A,\omega_B) \leftarrow \widetilde{\mathscr{G}}^{(e)}_{\hat{\rho}}\left(\overline{\omega}+\frac{\Omega}{2},\overline{\omega}-\frac{\Omega}{2}\right)$$



From 4D to (2+2)D in the frequency space

Electronic coherence, entanglement witness and scattering model

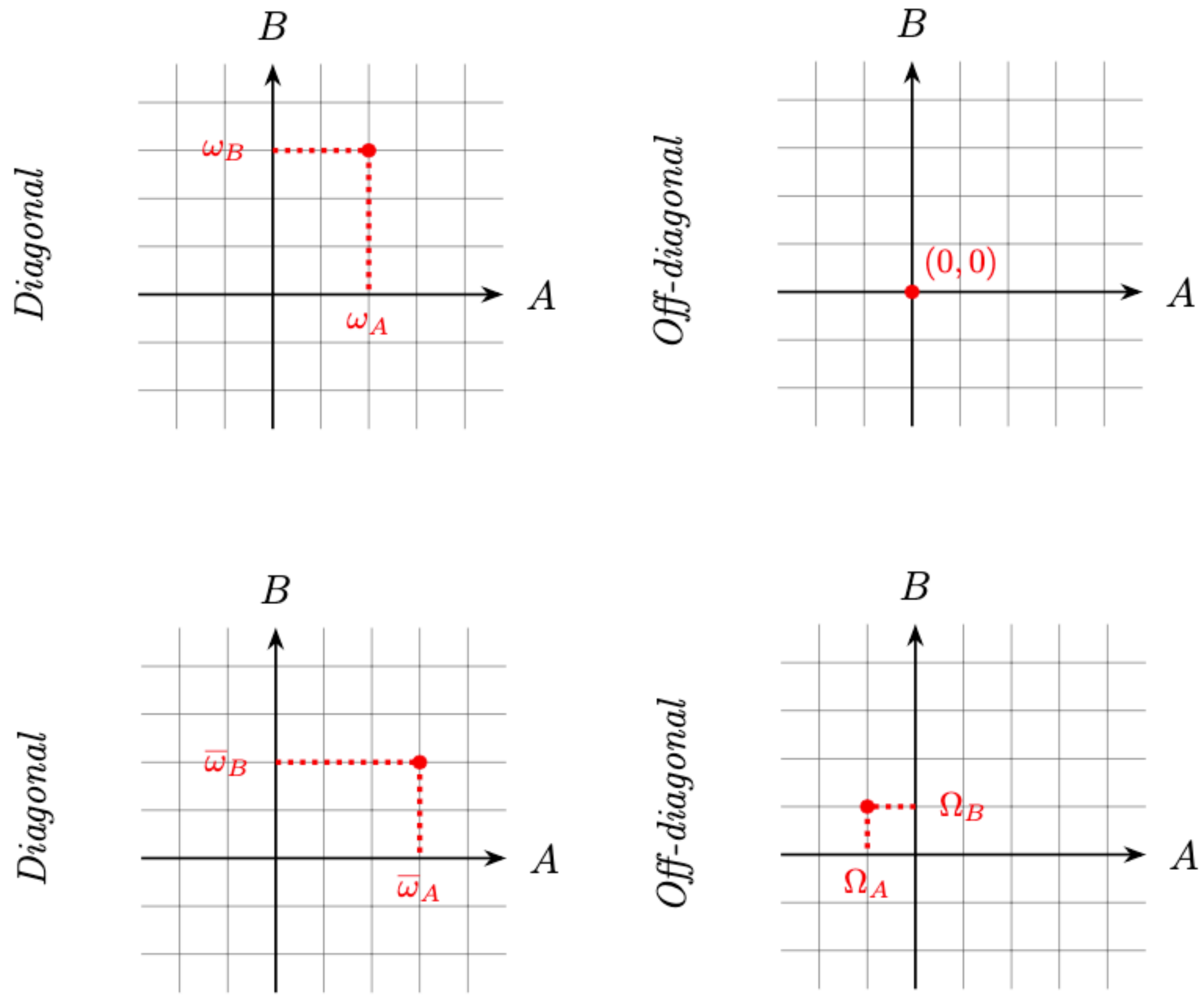
- Separation of the **4D frequency space** into **two 2D spaces**
  - Two « **diagonal** » dimensions ($\overline{\omega}_A$ and $\overline{\omega}_B$): **classical** plane
  - Two « **off-diagonal** » dimensions (Ω_A and Ω_B): **quantum** plane
- **Change** of variables: $\begin{cases} \overline{\omega}_A := \frac{\omega_A + \omega'_A}{2} \\ \Omega_A := \omega_A - \omega'_A \end{cases}$ and $\begin{cases} \overline{\omega}_B := \frac{\omega_B + \omega'_B}{2} \\ \Omega_B := \omega_B - \omega'_B \end{cases}$

$$\widetilde{\mathcal{G}}^{(2e)}_{\hat{\rho}}(\omega_A, \omega_B; \omega'_A, \omega'_B) \leftarrow \widetilde{\mathcal{G}}^{(2e)}_{\hat{\rho}}\left(\overline{\omega}_A + \frac{\Omega_A}{2}, \overline{\omega}_B + \frac{\Omega_B}{2}; \overline{\omega}_A - \frac{\Omega_A}{2}, \overline{\omega}_B - \frac{\Omega_B}{2}\right)$$

Examples in the (2+2)D frequency space

Electronic coherence, entanglement witness and scattering model

- **Plane waves** at energies ω_A and ω_B
  - $|\psi\rangle = \hat{c}_A^\dagger[\omega_A]\,\hat{c}_B^\dagger[\omega_B]\,|F\rangle$ with $|F\rangle := |F\rangle_A \wedge |F\rangle_B$
  - Perfectly localized $\Longrightarrow$ purely **classical**
- Superposition of **two Slater determinants**
  - $|\psi\rangle = \dfrac{1}{\sqrt{2}}\left(\hat{c}_A^\dagger[\omega_A]\,\hat{c}_B^\dagger[\omega_B] + \hat{c}_A^\dagger[\omega_A']\,\hat{c}_B^\dagger[\omega_B']\right)|F\rangle$
  - States **superposition** $\Longrightarrow$ **off-diagonal** terms



Cauchy-Schwarz inequality

Electronic coherence, entanglement witness and scattering model

- Cauchy-Schwarz **inequality**: $|\langle \mathbf{u}, \mathbf{v}\rangle|^2 \leq \langle \mathbf{u}, \mathbf{u}\rangle \langle \mathbf{v}, \mathbf{v}\rangle$
  - Scalar product on mathematical **operators**: $\langle \hat{A}, \hat{B}\rangle = \langle \hat{A}^\dagger \hat{B}\rangle_{\hat{\rho}} := Tr\big(\hat{\rho}\,\hat{A}^\dagger \hat{B}\,\big)$
  - Expression of the inequality: $|\langle \hat{A}_1\hat{A}_2\hat{B}_1\hat{B}_2\rangle_{\hat{\rho}}|^2 \leq \langle \hat{A}_1\hat{A}_1^\dagger\hat{B}_1\hat{B}_1^\dagger\rangle_{\hat{\rho}}\,\langle \hat{A}_2^\dagger\hat{A}_2\hat{B}_2^\dagger\hat{B}_2\rangle_{\hat{\rho}}$

- Apply to the **physical operators** $\left\{\hat{c}_A^\dagger[\omega_A], \hat{c}_A[\omega_A'], \hat{c}_B^\dagger[\omega_B], \hat{c}_B[\omega_B']\right\}$

$$|\,\widetilde{\mathcal{G}}_{\hat{\rho}}^{(2e)}(\omega_A, \omega_B; \omega_A', \omega_B')\,|^2 \leq \widetilde{\mathcal{G}}_{\hat{\rho}}^{(2e)}(\omega_A, \omega_B; \omega_A, \omega_B)\;\widetilde{\mathcal{G}}_{\hat{\rho}}^{(2e)}(\omega_A', \omega_B'; \omega_A', \omega_B')$$

Cauchy-Schwarz entanglement witness

Electronic coherence, entanglement witness and scattering model

- Cauchy-Schwarz « full-$\mathscr{G}^{(2e)}$ » **entanglement criterion** [Wölk, 2014]

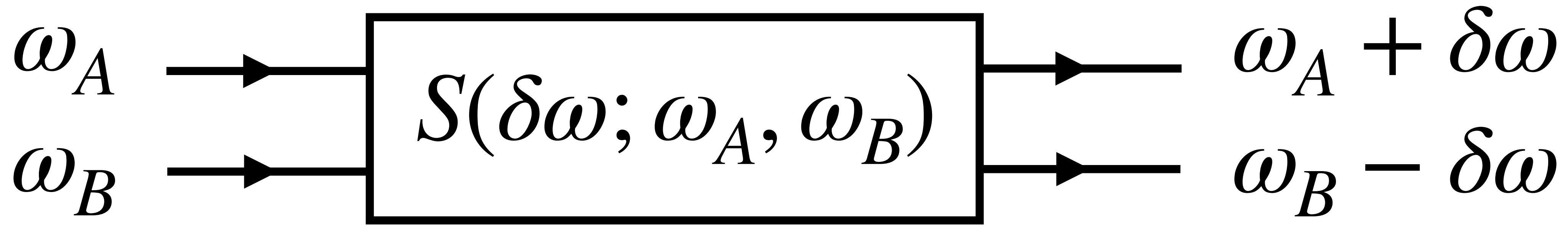
$$|\langle \hat{A}_1 \hat{A}_2 \hat{B}_1 \hat{B}_2 \rangle_{\hat{\rho}}|^2 \leq \langle \hat{A}_1 \hat{A}_1^\dagger \hat{B}_2^\dagger \hat{B}_2 \rangle_{\hat{\rho}} \, \langle \hat{A}_2^\dagger \hat{A}_2 \hat{B}_1 \hat{B}_1^\dagger \rangle_{\hat{\rho}}$$

- Respected by any **separable state**: can only be violated by **entangled states**
  - **Separable** state: $|\psi\rangle = \hat{O}_A \, \hat{O}_B \, |F\rangle$ (with $\hat{O}_{A/B}$ **creation operators** acting on states of $\mathscr{H}_{A/B}$)

$$|\, \widetilde{\mathscr{G}}_{\hat{\rho}}^{(2e)}(\omega_A, \omega_B; \omega'_A, \omega'_B) \,|^2 \leq \widetilde{\mathscr{G}}_{\hat{\rho}}^{(2e)}(\omega_A, \omega'_B; \omega_A, \omega'_B) \, \widetilde{\mathscr{G}}_{\hat{\rho}}^{(2e)}(\omega'_A, \omega_B; \omega'_A, \omega_B)$$

Two-electron coherent scattering model

Electronic coherence, entanglement witness and scattering model



- Two **electrons** propagate along **two channels** A and B with energies (ω_A, ω_B)
- **Scattering process** described by a **scattering matrix** $\hat{S}$
  - **Unitary** matrix $\hat{S}$, function of ω_A, ω_B, and $\delta\omega$
  - Limited bandwidth for the **exchanged** energy: $\delta\omega \in [-\omega_A, \omega_B]$
- **Coherent** scattering with **conservation** of the total energy: $E_{tot} = \omega_A + \omega_B$

II. Collision of two energy-localized wave-packets

Mathematical description

Collision of two energy-localized wave-packets

- Incoming electrons: **perfectly localized in energy** (at ω_A and ω_B)

$$|\psi\rangle_{in} := \hat{c}_A^\dagger[\omega_A]\,\hat{c}_B^\dagger[\omega_B]\,|F\rangle \quad \text{with} \quad |F\rangle := |F\rangle_A \wedge |F\rangle_B$$

- **Scattered** electrons: superposition of **all the possible energy transfers**

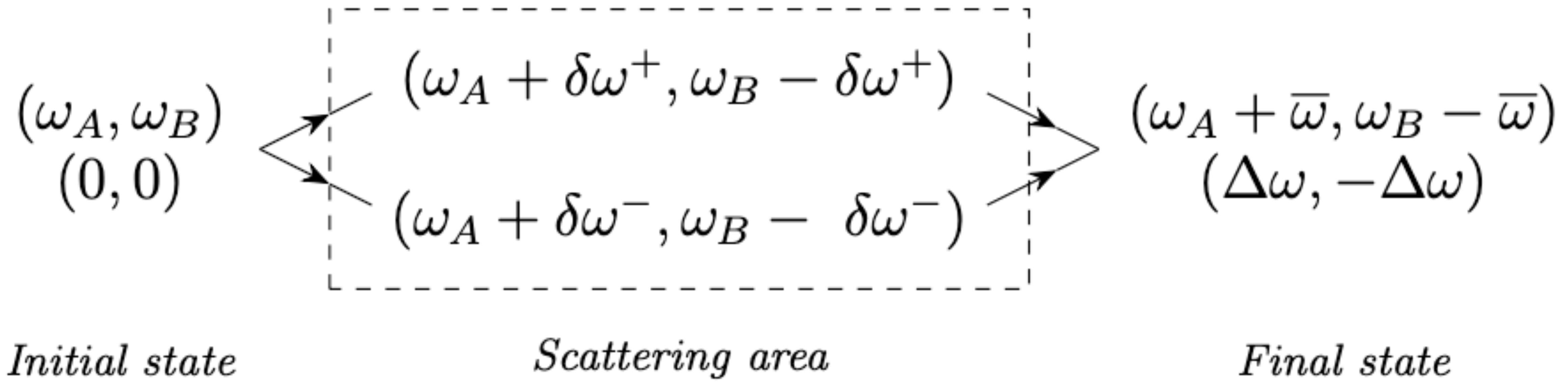
$$|\psi\rangle_{out} := \int_{\mathbb{R}} d(\delta\omega)\, S(\delta\omega; \omega_A, \omega_B)\, \hat{c}_A^\dagger[\omega_A + \delta\omega]\, \hat{c}_B^\dagger[\omega_B - \delta\omega]\, |F\rangle$$

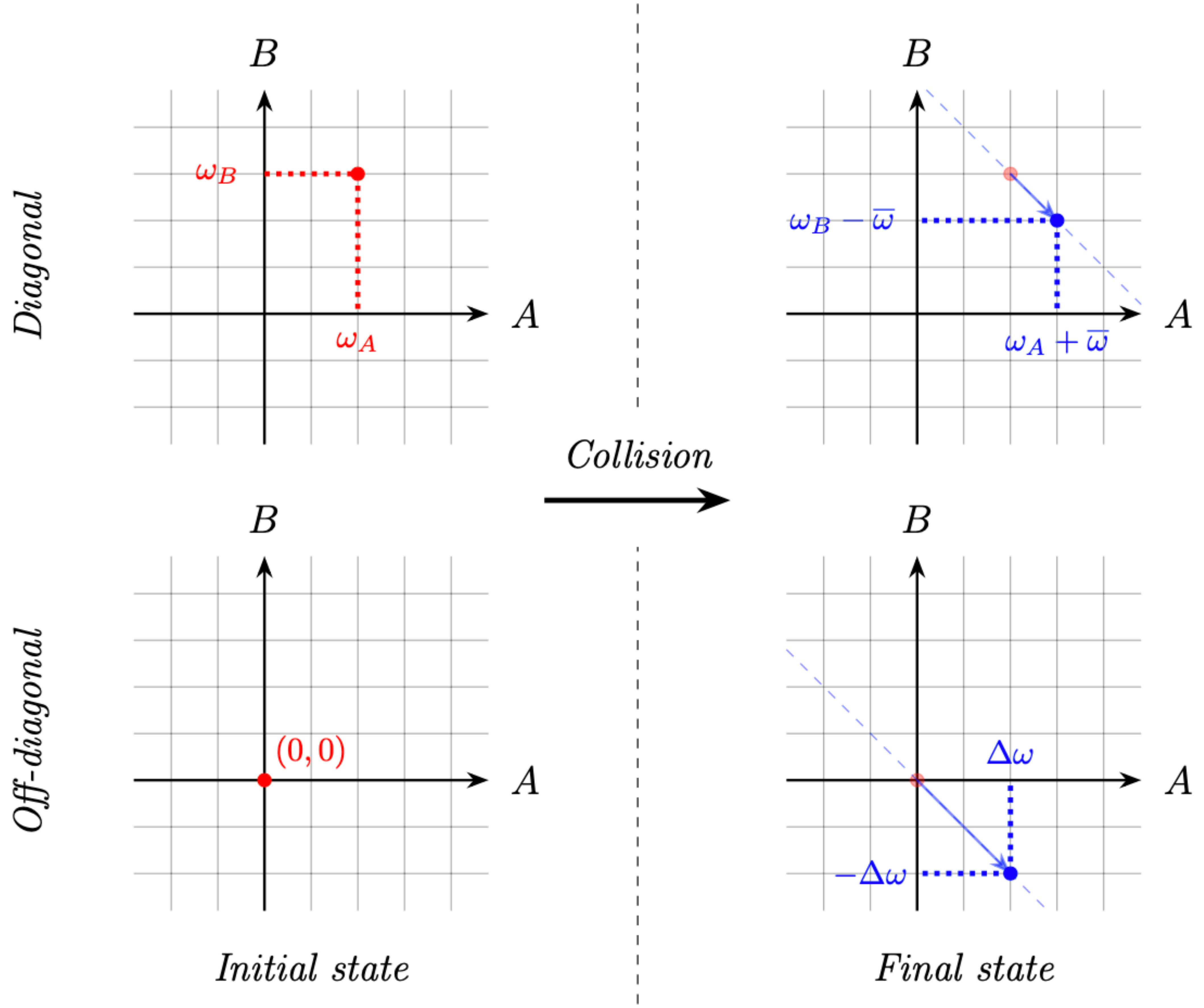
- **Goal of the study**: apply the full-$\mathcal{G}^{(2e)}$ criterion with $\hat{\rho}_{out} := |\psi\rangle_{out}\langle\psi|_{out}$

Representation of the scattering process

Collision of two energy-localized wave-packets

- Considering **two quantum paths**





- $\overline{\omega} := \dfrac{\delta\omega^+ + \delta\omega^-}{2}$ and $\Delta\omega := \delta\omega^+ - \delta\omega^-$

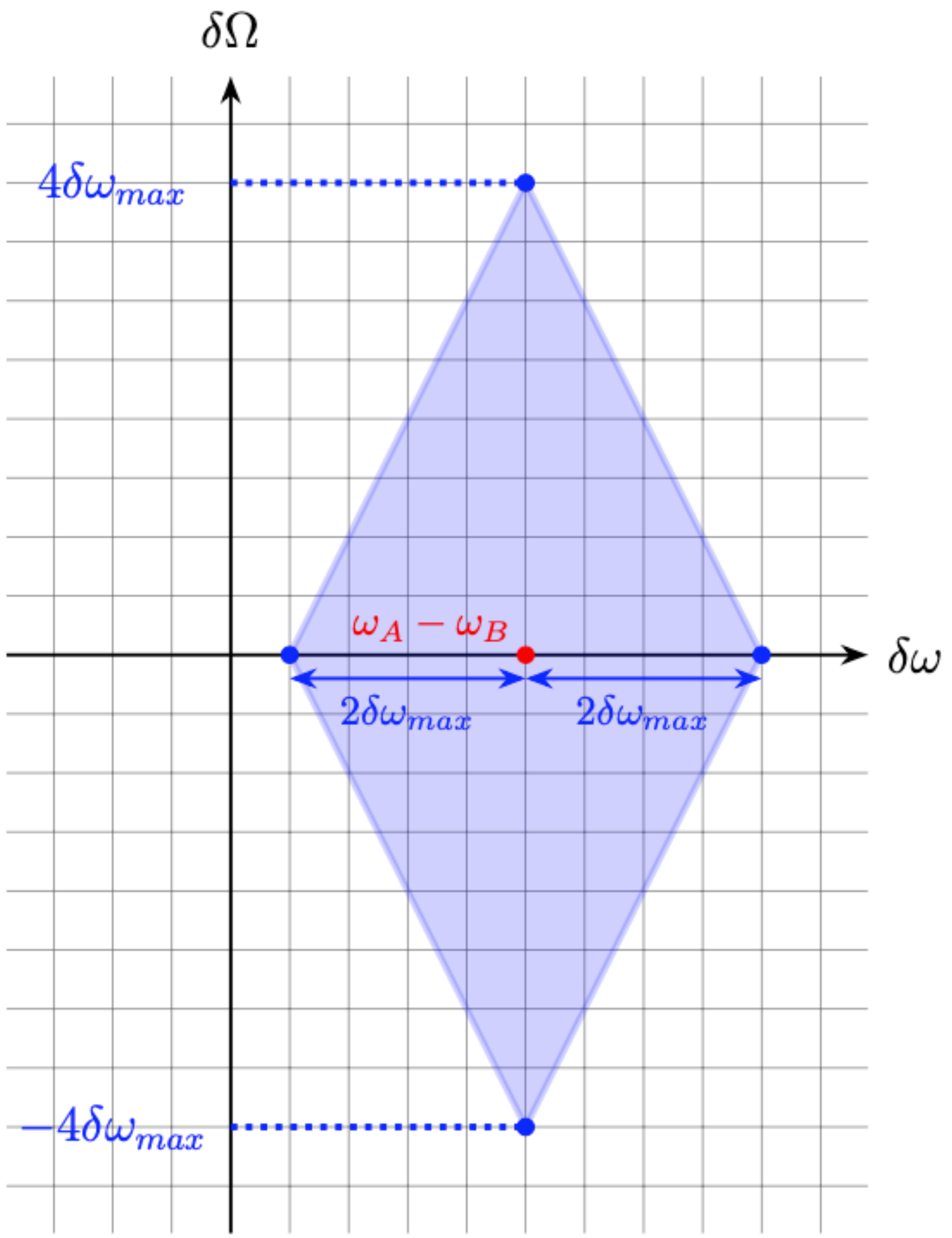
- **Conserved quantities:** $\omega_A + \omega_B$ and $\Omega_A + \Omega_B$

Visualization of the scattering process

Collision of two energy-localized wave-packets

- View in the plane $(\delta\omega, \delta\Omega) := (\omega_A - \omega_B, \Omega_A - \Omega_B)$
- Detection of both electrons **above the Fermi sea**
  - $|\delta\omega^{\pm}| \leq \delta\omega_{max}$: **rectangle** in the $(\delta\omega^+, \delta\omega^-)$ plane
  - **Rotation** and **rescaling** with $(\varpi, \Delta\omega)$

Scattered states represent a **diamond** in the $(\delta\omega, \delta\Omega)$ plane



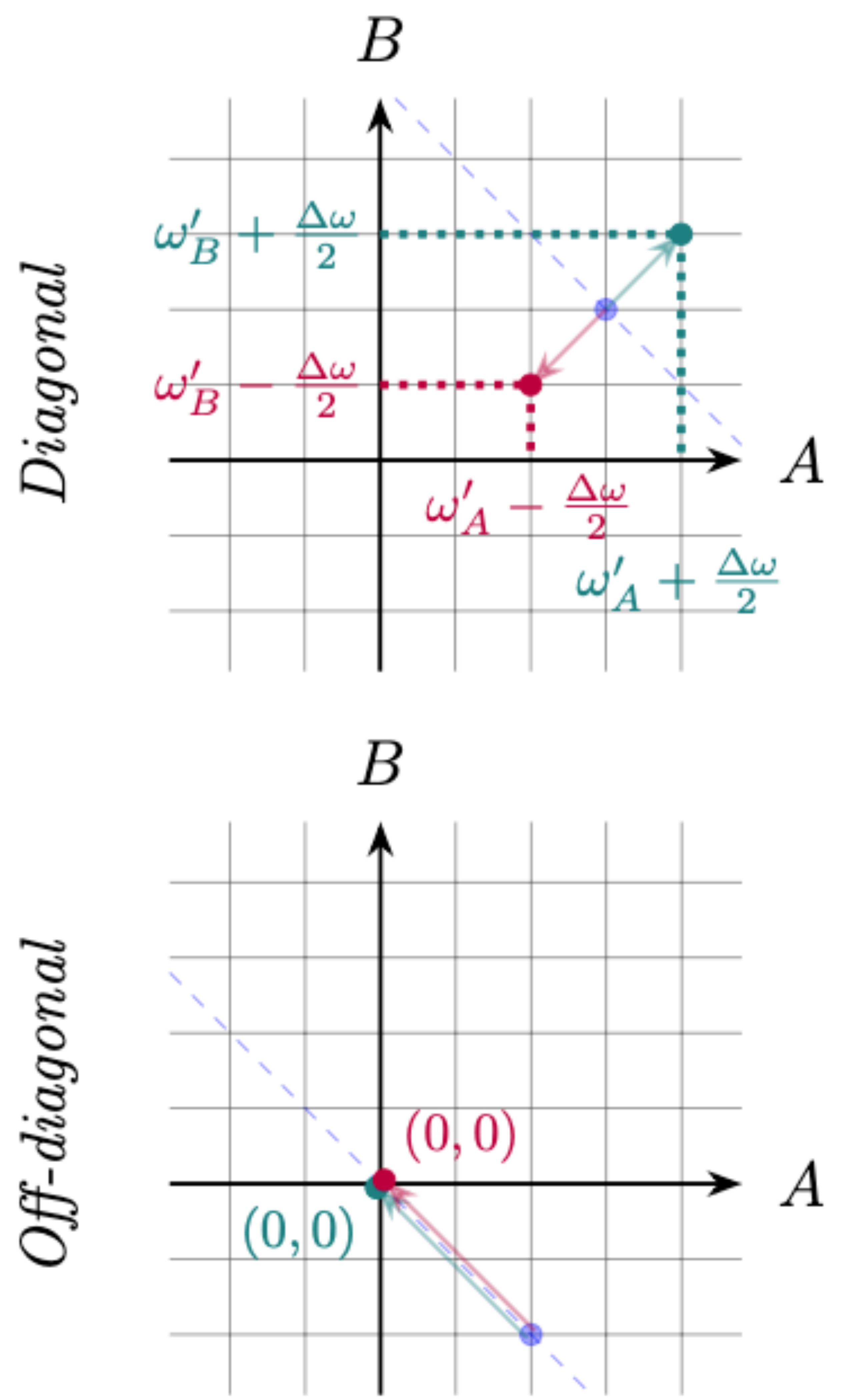
4D visualization of the entanglement witness

Collision of two energy-localized wave-packets

$$\left| \widetilde{\mathscr{G}}^{(2e)}_{\hat{\rho}}(\omega_A, \omega_B; \omega'_A, \omega'_B) \right|^2 \leq \widetilde{\mathscr{G}}^{(2e)}_{\hat{\rho}}(\omega_A, \omega'_B; \omega_A, \omega'_B)\, \widetilde{\mathscr{G}}^{(2e)}_{\hat{\rho}}(\omega'_A, \omega_B; \omega'_A, \omega_B)$$

- Coordinates of the **right-hand side** points
  - **Diagonal** component: **out of the energy conservation** line
  - **No off-diagonal** component: C.S. points are **always diagonal**
- **Witness violated** as soon as $\Delta\omega \neq 0$ ($\iff \delta\omega^+ \neq \delta\omega^-$)

The scattering process always creates entanglement



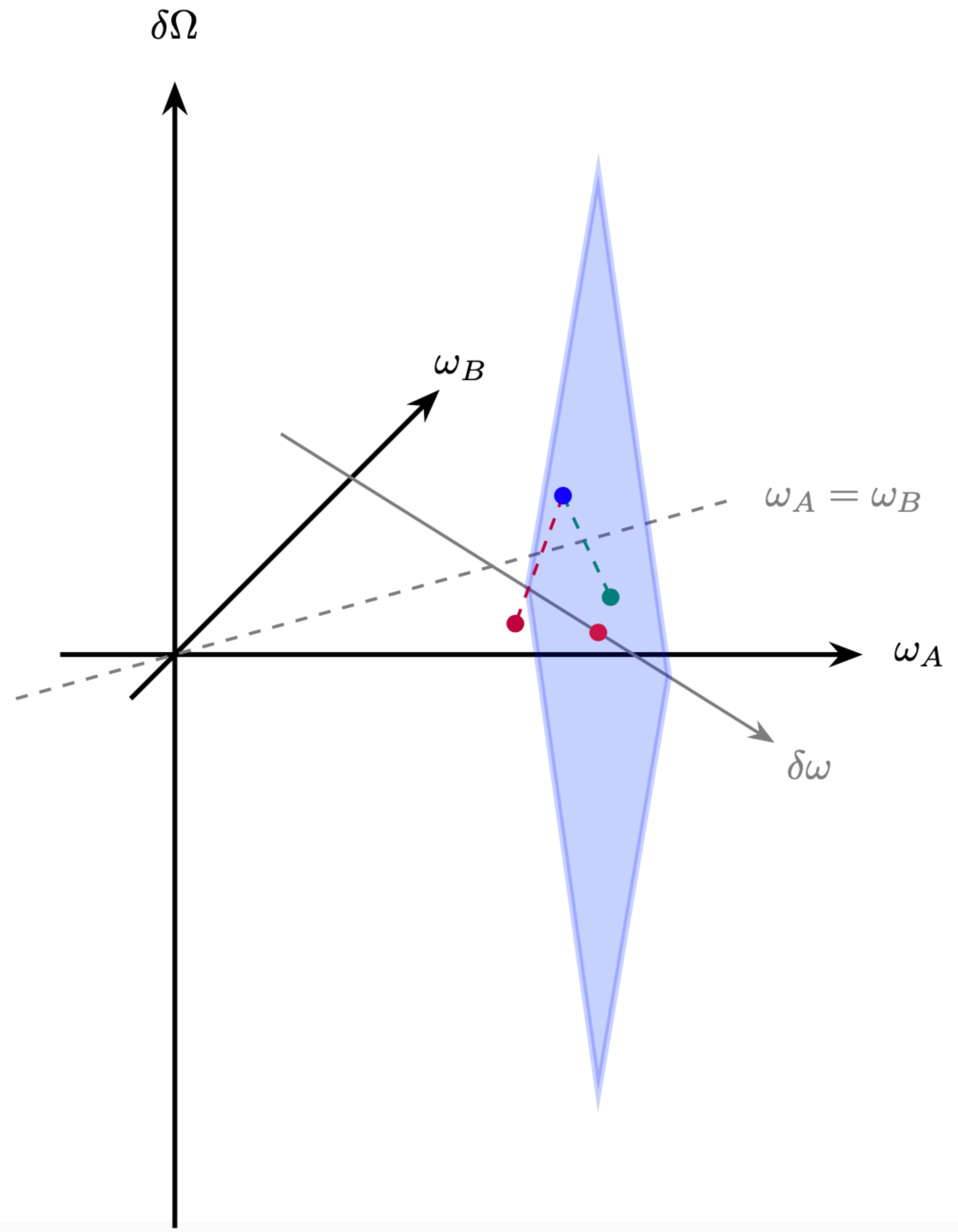
Entanglement witness: take-home messages

Collision of two energy-localized wave-packets

- Scattered states: « **flat** » **diamond** in the $(\delta\omega, \delta\Omega)$ plane
- **Expression in terms of $\hat{S}$ matrix coefficients:**

$$|S(\delta\omega^+; \omega_A, \omega_B)\, S(\delta\omega^-; \omega_A, \omega_B)^*|^2 \leq 0$$

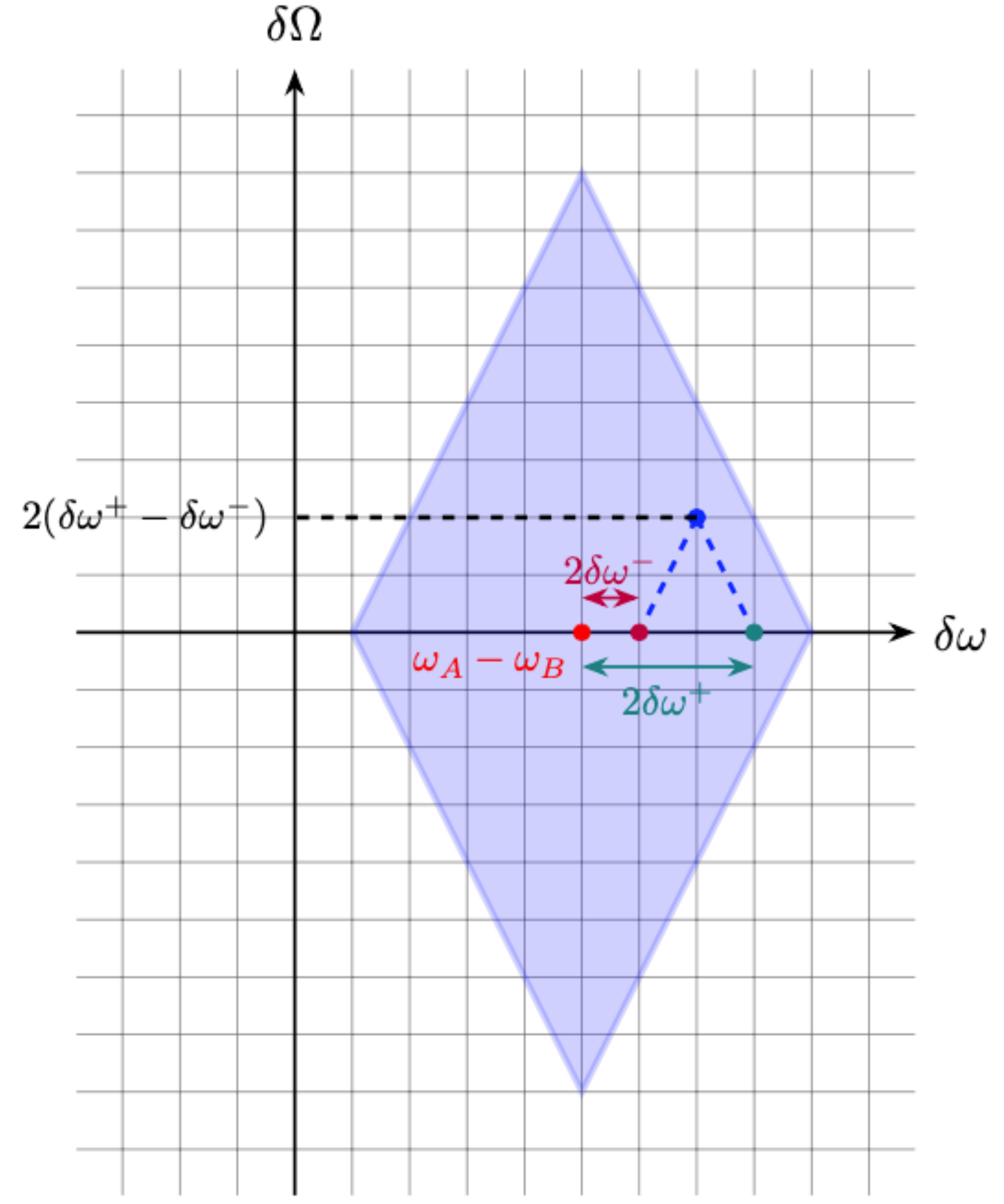
- **Scattering** $\Longrightarrow$ C.S. points **out of the diamond**
- **Witness necessarily violated if scattering**



Cauchy-Schwarz inequality's visualization

Collision of two energy-localized wave-packets

- **Any point of the diamond** is compared to **two points** on the diagonal line $\delta\Omega = 0$
- The inequality is **not trivially violated** as both comparison points are **always in the diamond**
- $(\delta\omega, \delta\Omega)$ is a convenient plane to visualize the Cauchy-Schwarz **inequality**



4D visualization of the C.S. inequality

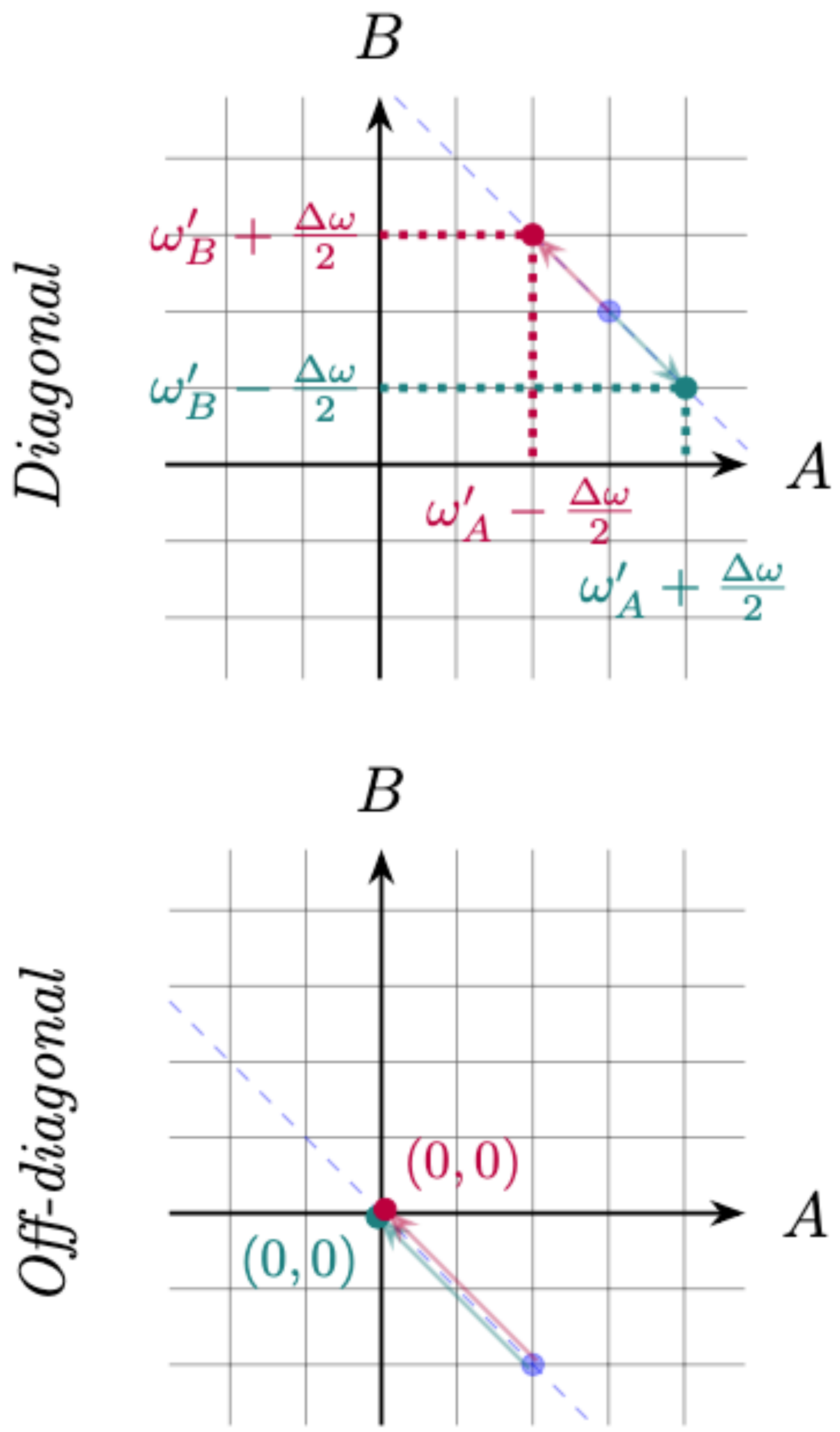
Collision of two energy-localized wave-packets

$$|\widetilde{\mathcal{G}}^{(2e)}_{\hat{\rho}_{out}}(\omega_A, \omega_B; \omega'_A, \omega'_B)|^2 \leq \widetilde{\mathcal{G}}^{(2e)}_{\hat{\rho}_{out}}(\omega_A, \omega_B; \omega_A, \omega_B)\, \widetilde{\mathcal{G}}^{(2e)}_{\hat{\rho}_{out}}(\omega'_A, \omega'_B; \omega'_A, \omega'_B)$$

- **Diagonal component:** along the **energy conservation** line
- Expression in terms of $\hat{S}$-matrix coefficients:

$$|S(\delta\omega^+; \omega_A, \omega_B)\, S(\delta\omega^-; \omega_A, \omega_B)^*|^2 \leq |S(\delta\omega^+; \omega_A, \omega_B)|^2\, |S(\delta\omega^-; \omega_A, \omega_B)|^2$$

- **Trivial equality**: never violated, respected by any scattered state



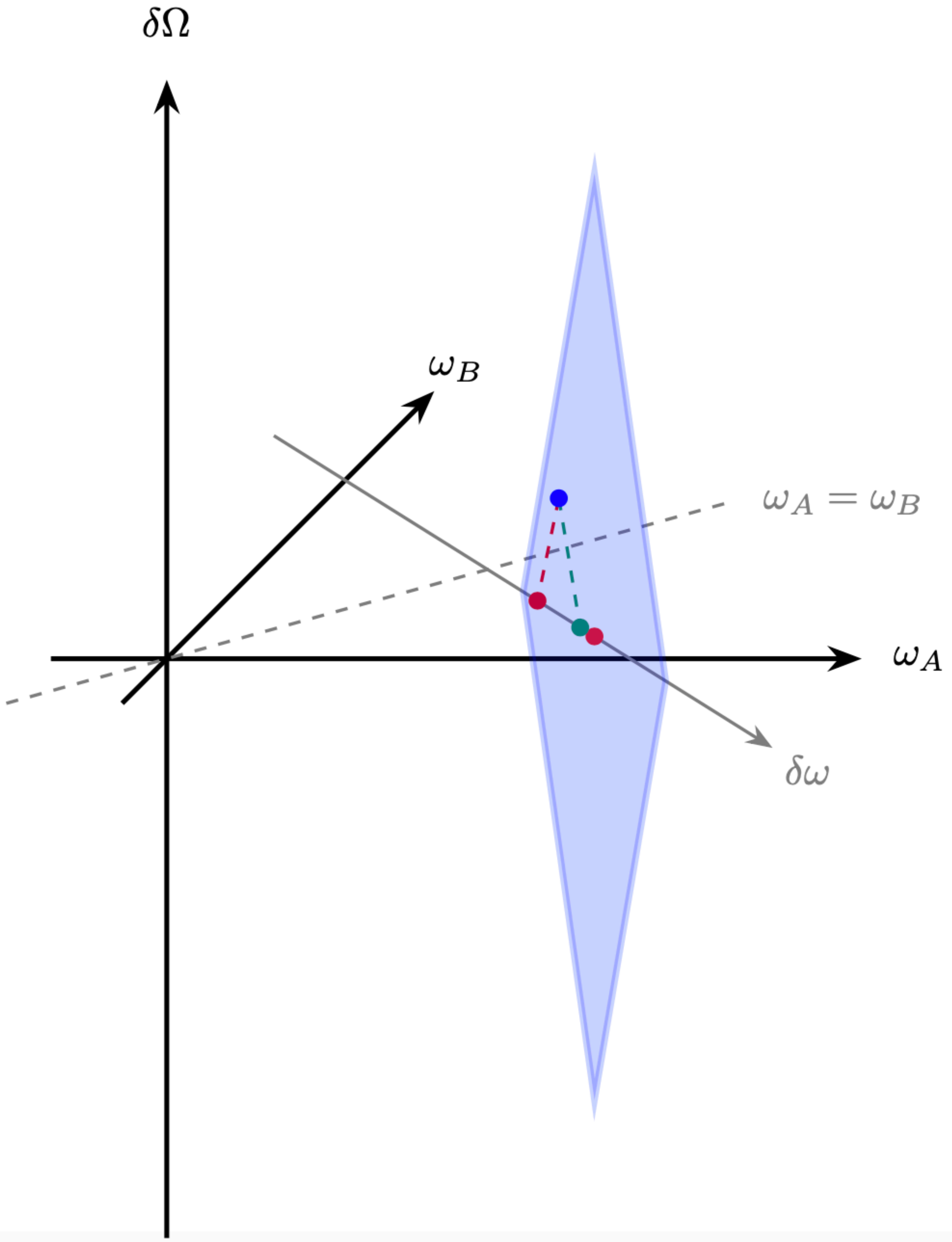
C.S. inequality: take home messages

Collision of two energy-localized wave-packets

- Cauchy-Schwarz bounds: **always in the diamond's plane**

- The inequality **becomes an equality**

Scattered states never violate the Cauchy-Schwarz inequality



III. Collision of two Landau wave-packets *(work in progress)*

Mathematical description

Collision of two Landau wave-packets

- Perfectly energy-localized wave-packets: **not realistic** (+ causes **divergences**)
- **Improvement** of the problem's description: **Lorentzian** wave-packets
  - Excitation in channel A/B: **Lorentzian energy distribution** of half-width $\frac{\gamma_{A/B}}{2}$ around $\omega_{A/B}$
  - **Wave-function** in the frequency space : $\widetilde{\varphi}_{\omega_{A/B}}(\omega) = \dfrac{\mathcal{N}_{A/B}\,\Theta(\omega)}{\omega - \omega_{A/B} - i\frac{\gamma_{A/B}}{2}}$
- **Single-electron** wave-packet: $|\varphi_\Omega\rangle := \hat{\psi}^\dagger[\varphi_\Omega]\,|F\rangle = \int_{\mathbb{R}} d\omega\, \widetilde{\varphi}_\Omega(\omega)\, \hat{c}^\dagger[\omega]\,|F\rangle$

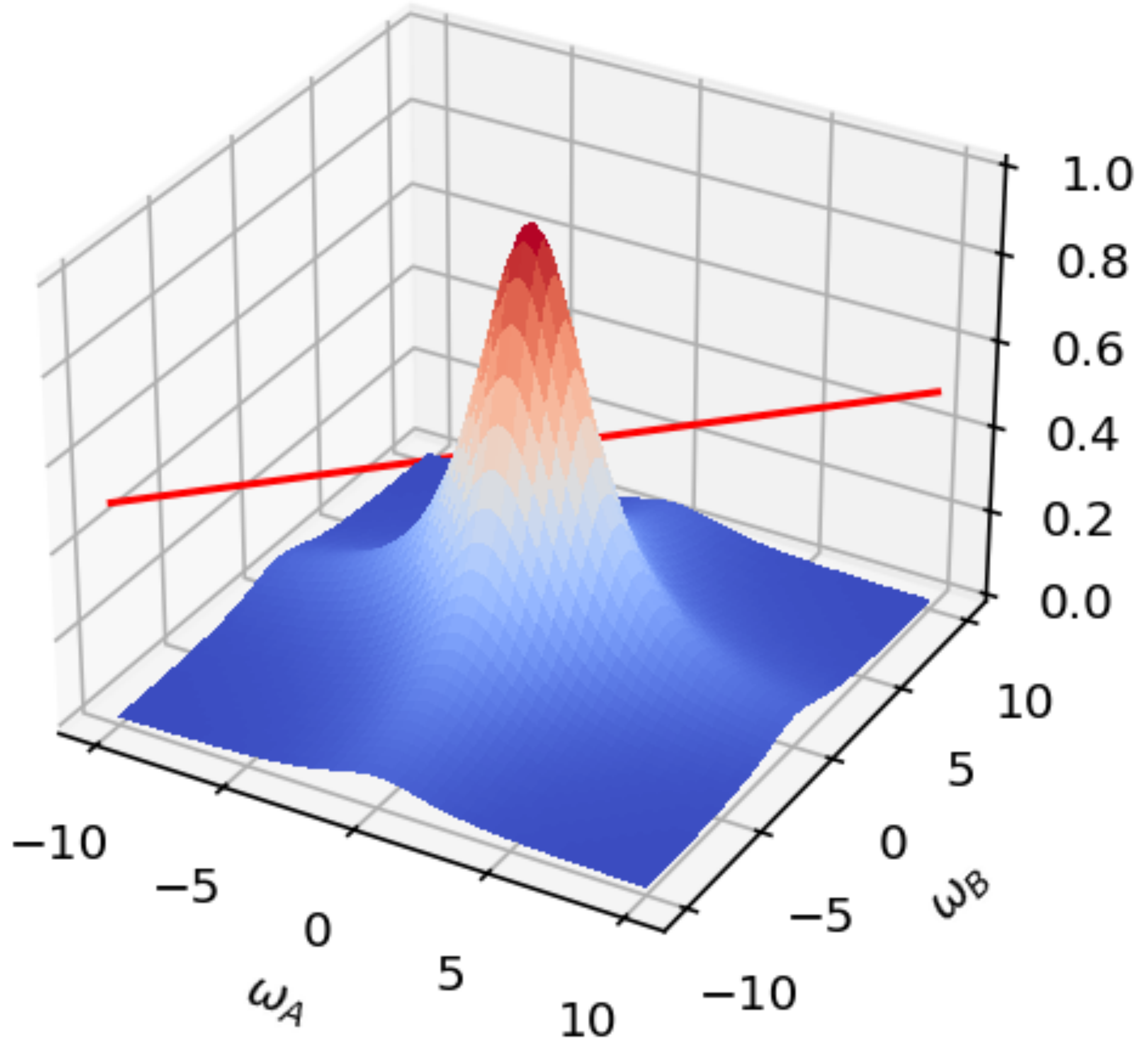
Wave-packet in the (ω_A, ω_B) space

Collision of two Landau wave-packets

- Two-electron wave-packet:

$$|\psi\rangle_{out} = \iiint_{\mathbb{R}^3} d\Omega\, d\omega_A\, d\omega_B\, S(\Omega; \omega_A, \omega_B) |\varphi_{\omega_A+\Omega}\rangle_A \wedge |\varphi_{\omega_B-\Omega}\rangle_B$$

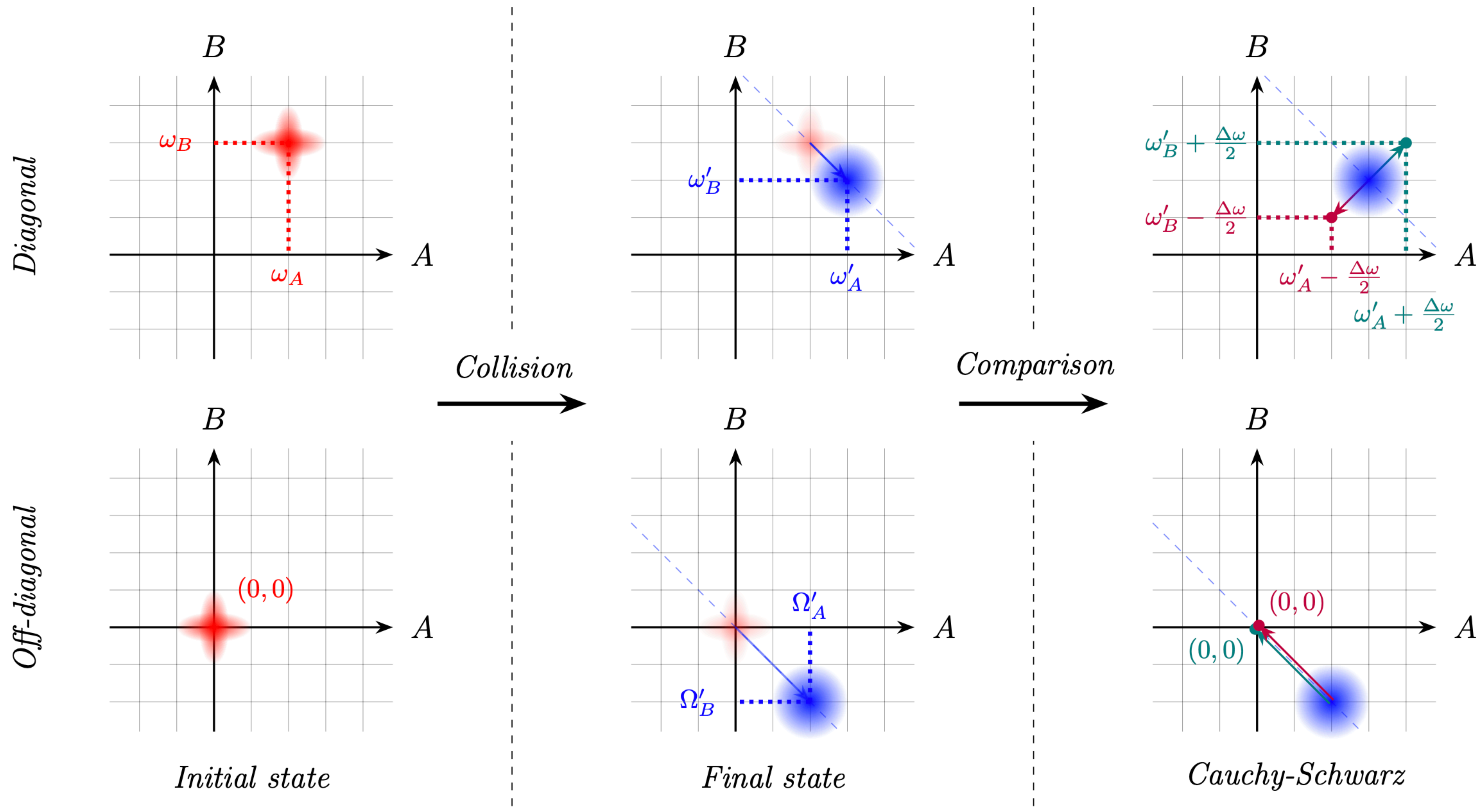
- 2D-Lorentzian: **width** in the $\omega_A = \omega_B$ direction?
  - $\gamma_A \simeq \gamma_B \ (:= \gamma)$: $\quad \gamma_{eff} \simeq \left(\frac{\sqrt{2}-1}{2}\right)^{1/2} \gamma \simeq 0.46\,\gamma$
  - $\gamma_{A/B} \gg \gamma_{B/A}$: $\quad \gamma_{eff} = \frac{\min(\gamma_A, \gamma_B)}{\sqrt{2}}$



Intuition on the results

Collision of two Landau wave-packets

- **Spreading** of the wave-packet
- The diamond is « **spread** » in the $\omega_A = \omega_B$ direction
- Witness **respected**: **overlap** between the scattered state and the C.S. points



What happens next?

Collision of two Landau wave-packets

- **Express the witness** for Landau wave-packets and **check the results**
- Consider a more **realistic / general** model
    - **Incoherent** scattering: **energy exchanged with the environment** during the interaction
    - Specify the interaction: **radiation coupler** without total mutual influence
    - Leads to an **expression** for the $\hat{S}$-matrix: convenient for **quantitative predictions**
- Study **Levitonic** wave-packets (Lorentzian **temporal** distribution)

Thank you for your attention!